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EROSION IN RADIAL INFLOW TURBINES - VOLUME V: COMPUTER PROGRAMS FOR TRACING PARTICLE TRAJECTORIES

By:

W.B. Clevenger, Jr.

and

W. Tabakoff

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16. Abstract

This report presents the computer programs that have been used to study the trajectories of particles in the radial inflow turbines. Included are descriptions of the general technique that is followed by each of the programs. A set of subroutines that have been developed during the study are described. Descriptions, listings, and typical examples of each of the main programs are included.

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SUMMARY

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INTRODUCTION

Particle erosion in gas turbine engines has become important because significant decreases in the operating life and rated performance have resulted when these engines are used in dusty environments. For example, the engines in military helicopters, operating at low altitudes and remote landing fields, have significantly shorter life and a more rapid performance deterioration rate than engines operating from hard surfaced landing fields and at higher altitudes. Although these helicopters have main engines which utilize axial flow turbines, some also have auxiliary power sources for special devices which utilize radial turbines, as shown schematically in Figure 1. Radial inflow turbines have also been used on small portable power plants which are also likely to be used in areas where dust ingestion will occur. Radial inflow turbines also are seriously being considered for future use in advanced helicopter engines and transportation vehicles such as trucks, buses, and automobiles. These engines will at times have incomplete filtering of incoming air, leading to the ingestion of erosive-size particles that could seriously degrade engine performance.

The radial turbine engines have, however, a more serious erosion problem than axial-flow turbines. In radial turbines, the heavier particles can experience a radially outward centrifugal force that is greater than the radially inward component of the aerodynamic drag force. In the axial-flow turbine, the centrifugal force acts perpendicular to the aerodynamic drag force. Thus, in radial turbines the heavier particles can be trapped between the stator and rotor, resulting in the particles striking the trailing edges of the stators and the leading edges of the rotor many times. In axial-flow turbines, the particles generally move outward to the tip region, but all particles have a tendency to pass through the turbines.

As a result of the wide interest in using radial turbines and because erosion seems to be more severe in radial turbines, an investigation was sponsored by the NASA Lewis Research Center

to study erosion phenomena in radial inflow turbines and to find ways of eliminating this erosion. Included in this investigation are analytical studies of particle trajectories through a radial inflow turbine and predictions of the effect of blade materials erosion by the action of particles. The results of these investigations will be published in a series of five volumes.

Volumes I through III (1, 2, 3) have dealt with the concept of erosive particle trajectories and studied the approximate velocities and types of impacts that occurred on surface inside a radial turbine as particles moved through the turbine.

Volume III (3) indicated several possible problem areas within a radial turbine. The first is the region at the end of the scroll, where the more rapidly changing radius of curvature causes more moderate angle impacts by the particles. Because of the nature of the motion of particles in the scroll, those particles that do not immediately enter the stator will tend to accumulate and enter a few of the blade passages near the scroll exit. Volume III also revealed that most heavier particles will become trapped in the vortex region of the turbine and repeatedly strike the trailing edges of the stator and the rotor leading edge.

Volume IV (4) presented an analtyical study of the rate at which material is removed by ingested dust impinging on the internal surfaces of a typical radial turbine. The study indicates that there are several regions which experience very severe erosion loss, and other regions that experience moderate levels of erosion loss.

The purpose of this report (Volume V) is to present the computer programs that have been developed to trace the particle trajectories through the various parts of the radial inflow turbine. The programs will be useful for any future study of the erosion phenomena that might be considered for a given turbine. Several studies involving computer programs are cited within this report. These programs have been used to determine the gas flow solutions in a given geometry and do not offer a comparison with the trajectory trace programs.

A set of the program source decks on tape is available from COSMIC (Computer Software Management and Information Center), Computer Center, University of Georgia, Athens, Georgia 30601. The programs can be ordered by using the number of this Report as Identification.

GENERAL NUMERICAL TECHNIQUE

Main Programs

Much of the analytical work that has been done in this series of reports has required the numerical solution of the ordinary differential equations that describe the motion of a particle in a gas flow field. These equations are derived in Volume III (3); and their final form which will be integrated numerically is given as Equation (17) in the same reference. These equations are expressed as a set of first order ordinary differential equations by assuming

$$\begin{aligned} y_1 &= r \\ y_2 &= \delta r/\delta t \\ y_3 &= \theta \\ y_4 &= \delta\theta/\delta t \\ y_5 &= z \\ y_6 &= \delta z/\delta t \end{aligned} \tag{1}$$

The equations of motion can then be expressed as

$$\frac{\delta y_1}{\delta t} = y_2$$

$$\frac{\delta y_2}{\delta t} = y_1 y_4^{\,2} + 2y_1 y_4 \dot{\omega} + y_1 \omega^2 + B|v|v_r - g\cos(\theta-\sigma)$$

$$\frac{\delta y_3}{\delta t} = y_4$$

$$\frac{\delta y_4}{\delta t} = \frac{1}{y_1}\left[-2y_2y_4 - 2\omega y_2 + B|v|v_\theta + g\sin(\theta+\sigma)\right]$$

$$\frac{\delta y_5}{\delta t} = y_6$$

$$\frac{\delta y_6}{\delta t} = B|v|v_z - g\sin\phi \qquad (2)$$

Where $B = \frac{1}{2}\frac{C_D A}{m} = \frac{3}{4}\frac{C_D}{\rho_p D_p}$ for spherical particles. The terms V_r, V_θ and V_z represent the velocity of the gas relative to the moving particle.

$$V_r = w_r - y_2$$

$$V_\theta = w_u - y_1 y_4$$

$$V_z = w_z - y_6 \qquad (3)$$

In the solution of these equations, a fourth order Runge-Kutta integration technique, as described in Reference 5 was used. The form of the integration formula is

$$y_{i+1,j} = y_{i,j} + \frac{h}{6}(k_{1j} + 2k_{2j} + 2k_{3j} + k_{4j}) \qquad (4)$$

for the differential equation $\frac{dy}{dx} = f(x,y)$

where $k_{1j} = f_j(x_i, y_i)$

$$k_2 j = f_j\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}h\,k_{1j}\right)$$

$$k_{3j} = f_j(x_i + \tfrac{1}{2} h,\ y_i + \tfrac{1}{2} h\, k_{2j})$$

$$k_{4j} = f_j(x_i + h,\ y_i + h\, k_{3j}) \qquad (5)$$

The values of the function f_j corresponding to the equations of motion (2) are:

$$f_1 = v_2$$

$$f_2 = y_1 y_4^2 + 2y_1 y_4\, \omega + y_1 \omega^2 + B|V|V_r - g \cos(\theta - \sigma)$$

$$f_3 = y_4$$

$$f_4 = \frac{1}{y_1} [-2y_2 y_4 - 2\omega y_2 + B|V|V_\theta + g \sin(\theta + \sigma)]$$

$$f_5 = y_6$$

$$f_6 = B|V|V_z - g \cdot \sin \phi$$

In Equation (4), h represents the time increment between integration steps. In most of the work that is presented here, a value of 1×10^{-5} seconds was used. One of the longest trajectories in the scroll was calculated with different time increments to study the deviation in trajectories as the time increment decreased. The results of this numerical experiment are presented in Figure 2 which shows the variations in the angular position of the particle when different time increments were used. At the smaller time steps, the round off errors can be reduced by switching to double precision. As the time increment increases, the truncation errors can only be reduced if a higher order numerical procedure is considered. In this example, the trajectory is approximately 100 cm long and the deviation of the trajectory by one degree would correspond to a linear distance of 0.3 cm. In most of the regions in the

turbine, the total distance traveled by the particle is much shorter than the distances indicated here; so the corresponding error in the position of the particle would also be much smaller. Also indicated in Figure 2 is the amount of central processing unit (CPU) time that was required for the various solutions. As the time increment decreases, the computational time required to complete the solution increases.

In the programs that were used to calculate the gas flow field, the magnitude and direction of the velocities were determined at the grid points. A linear interpolation technique was used to estimate the magnitude and direction of the gas velocity at the points within the grid. An iteration scheme was used to calculate average gas flow field properties which were needed in the particle trajectory calculations at each time increment. Figure 3 illustrates the general procedure that was followed in this iteration scheme. After integrating the equation of motion over the time segment, the magnitude and direction of the gas velocity and the gas properties at the new particle location are found and used to calculate an improved estimate of the average velocity components and average gas properties. The program compares the new estimated average values with the old average values; and, if necessary, reintegrates the equations of motion to find a corrected new location for the particle. Usually the procedure converges in only a few iterations. In a few cases where the particle size was quite small, the procedure failed to converge in 100 iterations. When a smaller time increment was used, the solution could be found beyond the point where the iteration technique failed to converge.

Generally, all the programs listed in this report will calculate several similarity parameters that are useful in relating different particles that have similar trajectories. These similarity parameters are explained in greater detail in Volume I (1) of this series of reports, but it is worthwhile to define the terms that are calculated by the programs.

The characteristic length as calculated in the programs is

$$\delta = \frac{10}{3} \frac{\rho_p D_p}{\rho_g^*} \tag{7}$$

This parameter can be used to relate trajectories of different particles in equivalent flow fields as long as the Reynolds number of the particle is generally greater than 500. If the Reynolds number drops below 500, the two particles may not follow precisely the same trajectory, but the trajectories will not be significantly different.

The time constant as calculated by the programs is

$$\tau = \frac{\rho_p D_p^2}{18 \mu_g^*} \tag{8}$$

This parameter generally applies to very small particles because its application is restricted to cases where the particles Reynolds number is less than 1.

An approximate Reynolds number is calculated using critical gas flow properties and one half the gas velocity. This term calculated by the programs is

$$Re_{cr} = \frac{\rho_g^* (V_{cr}/2.) D_p}{\mu_g^*} \tag{9}$$

Although this term was retained in the programs, experience has shown that is is not necessarily typical of the particles Reynolds number.

Several special duty subroutines were used in conjunction with the main program to solve the particle trajectory. The remainder of this section will describe the subroutines that were used. In some cases, slightly different versions of a subroutine were used because of different types of geometry that occur between cases. The source deck listings of the programs contain the main program and all required subroutines.

Subroutine BOUNCE

The subroutine BOUNCE can be used in any program that deals with the trajectories of particles in a flow field where it is necessary to allow the particles to bounce off a surface. Generally, the main program will trace the particle trajectory and, at each new point along the trajectory, the main program should check the position of the particle. When the new position of the particle is found to be outside the boundaries of the contour surrounding the flow field, the main program specifies in ordered arrays the trajectory characteristics of the particle and the orientation of the boundary surface near the location where the particle impact occurs. The main program then calls BOUNCE, which determines the location where the particle trajectory intersects the surface and the time increment required for the particle to travel from the last point of the completed integration to the surface. The subroutine then returns the bounce point, and the time corresponding to the particle motion away from the bounce point. Figure 4 illustrates the surface, particle trajectory intersection.

Referring to Figure 4, the point $\bar{P}$ is the last point along the trajectory that is still in bounds and point $\bar{\bar{P}}$ is the next point along the trajectory that is out of bounds. Since these two points are vector quantities within the program, the particle velocity is calculated from the following equation:

$$\bar{V} = (\bar{\bar{P}} - \bar{P})/h \qquad (10)$$

Where h is the time increment used in the integration step.

The description of the surface near the location of the bounce is done in the main program by specifying three points that lie on the surface. These points are indicated by the position vectors $\bar{A}$, $\bar{B}$, and $\bar{C}$ within the subroutine. The subroutine then determines vectors $\bar{\bar{A}}\bar{B}$ and $\bar{\bar{A}}\bar{C}$ which lie in the surface, where

$$\bar{AB} = \bar{B} - \bar{A} \tag{11}$$

$$\bar{AC} = \bar{C} - \bar{A} \tag{12}$$

Next the subroutine determines the unit vector normal to the surface by calculating the cross product of $\bar{AB}$ and $\bar{AC}$, and then dividing by the magnitude of the resulting vector. Indicated below is the equation form of this operation.

$$\bar{AB} \times \bar{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \end{vmatrix} \tag{13}$$

$$\begin{aligned} &= (AB_2\ AC_3 - AB_3\ AC_2)\ \bar{i} \\ &\quad + (AB_3\ AC_1 - AB_1\ AC_3)\ \bar{j} \\ &\quad + (AB_1\ AC_2 - AB_2\ AC_1)\ \bar{k} \end{aligned} \tag{14}$$

$$\bar{UN} = \frac{\bar{AB} \times \bar{AC}}{|\bar{AB} \times \bar{AC}|} = UN_1\ \bar{i} + UN_2\ \bar{j} + UN_3\ \bar{k} \tag{15}$$

If $|\bar{AB} \times \bar{AC}| = 0$, the subroutine returns NFIX = 0 . This corresponds to a case where points $\bar{A}$, $\bar{B}$, and $\bar{C}$ lie on a straight line and hence are not sufficient to describe a surface.

The next step in the solution of the bounce problem is the determination of the point, $\bar{PB}$, where the particle bounces off the surface. This is determined by solving simultaneously the equations of the plane given by the unit vector $\bar{UN}$ and the point $\bar{A}$ and the straight line connecting the points $\bar{P}$ and $\bar{PP}$ of the trajectory. The equation of the plane is

$$UN_1(x-A_1) + UN_2(y-A_2) + UN_3(z-A_3) = 0 \tag{16}$$

The equations for the line trajectory are

$$\frac{x-P_1}{V_1} = \frac{y-P_2}{V_2} = \frac{z-P_3}{V_3} \tag{17}$$

These can be arranged into a set of three equations with three unknowns.

$$\begin{aligned} UN_1\, x + UN_2\, y + UN_3\, z &= D_1 \\ V_2\, x - V_1\, y &= D_2 \\ V_3\, x - V_1\, z &= D_3 \end{aligned} \tag{18}$$

where

$$\begin{aligned} D_1 &= UN_1A_1 + UN_2A_2 + UN_3A_3 \\ D_2 &= V_2P_1 - V_1P_2 \\ D_3 &= V_3P_1 - V_1P_3 \end{aligned} \tag{19}$$

The solution of these equations does not exist when the coefficient determinant is zero. The coefficient determinant is:

$$\begin{vmatrix} UN_1 & UN_2 & UN_3 \\ V_2 & -V_1 & 0 \\ V_3 & 0 & -V_1 \end{vmatrix} = UN_1V_1^2 + UN_2V_1V_2 + UN_3V_1V_3 \tag{20}$$

The coefficient determinant will be zero in the special cases that are outlined below.

Case 1. If $UN_1 = 0$, $V_2 = 0$, $V_3 = 0$, then the surface is parallel to the x axis and the particle motion is also parallel to the x axis. This event could not cause a bounce to occur.

Case 2. If $UN_1 = 0$, $UN_2 = 0$, and $V_3 = 0$, then the surface is parallel to the (x,y) plane and the particle motion is also parallel to the (x,y) plane, then the particle does not bounce.

Case 3. If $UN_1 = 0$, $UN_3 = 0$, and $V_2 = 0$, then the surface is parallel to the (x,y) plane and the particle motion is also parallel to the (x,y) plane, and no particle bounce takes place.

Case 4. If $V_1 = 0$, then the x coordinate of P and PB are the same. The solution of the intersection problem then requires use of only one of the equations for the line segment. The new equation set is

$$\begin{aligned} UN_2\ y + UN_3\ z &= D_4 \\ V_3\ y - V_2\ z &= D_5 \end{aligned} \tag{21}$$

where

$$\begin{aligned} D_4 &= D_1 - UN_1 \cdot P_1 \\ D_5 &= V_3 P_2 - V_2 P_3 \end{aligned} \tag{22}$$

The solution of this system of equations, with $PB_1 = P_1$ provides the intersection point. In this case, the determinant can be written as

$$(UN_2\ V_2 + UN_3\ V_3) = 0 \tag{23}$$

It can be equal to zero if $V_2 = 0$, and $UN_3 = 0$ in which case the surface is parallel to the (x,y) plane and the particle moves in the direction parallel to the z axis. Thus, $PB_2 = P_2$ and $PB_3 = A_3$. This may also occur when $V_3 = 0$, or $UN_2 = 0$ in which case the surface is parallel to the (x,y) plane and the particle moves parallel to the y axis. Then $PB_2 = A_2$, and $PB_3 = P_3$. All cases have been considered so far, since the three velocity components V_1, V_2 and V_3 cannot be all equal to zero.

The subroutine next calculates the distances between $\bar{PB}$ and $\bar{P}$, $\bar{PP}$ and $\bar{P}$, and $\bar{PB}$ and $\bar{PP}$. These distances can be expressed in equation form as

$$\begin{aligned} DPB &= |\bar{PB} - \bar{P}| \\ DPP &= |\bar{PP} - \bar{P}| \\ DPPB &= |\bar{PB} - \bar{PP}| \end{aligned} \tag{24}$$

Figure 5 illustrates a situation that can occur sometimes near a surface. If the main program specified points on surface 2, the incorrect bounce point is determined as indicated by the dashed extension of the trajectory. Similarly, referring to Figure 6, the incorrect bounce point which is obtained if the points on surface 1 are specified in the main program. This type of error is avoided by consistently using surface 1. The distance DPP = $|\bar{PP} - \bar{P}|$ is calculated and the subroutine checks to insure if it is greater than the distance DPPB = $|\bar{PP} - \bar{PB}|$. If DPP < DPPB, then the subroutine returns NFIX = 2. The main program should be written to recognize this and to switch to the appropriate surface.

The subroutine next uses one of two methods to determine the velocity components of the particle as the particle bounces from the surface. Method 1 is used when DPB $\geq \frac{1}{2}$ DPP. In this case, the particle travels over more than half the time segment before the impact occurs. Figure 7 illustrates the technique that is used. The first step in the subroutine is to determine the point $\bar{PN}$ which is the normal projection of the point $\bar{P}$ onto the surface. This is found by solving the equation of the plane, Equation (16), and the equations of the line that passes through the point $\bar{P}$ and is normal to the surface:

$$\frac{x-P_1}{UN_1} = \frac{y-P_2}{UN_2} = \frac{z-P_3}{UN_3} \tag{25}$$

The following set of three equations are obtained from the above relations and the equation of the plane:

$$\begin{aligned} UN_1\, x + UN_2\, y + UN_3\, z &= D_1 \\ UN_2\, x - UN_1\, y &= D_2 \\ UN_3\, x - UN_1\, a &= D_3 \end{aligned} \tag{26}$$

where

$$D_1 = UN_1\ A_1 + UN_2\ A_2 + UN_3\ A_3$$

$$D_2 = UN_2\ P_1 - UN_1\ P_2$$

$$D_3 = UN_3\ P_1 - UN_1\ P_3 \tag{27}$$

The solution of this system of equations gives the desired position vector $\bar{PN}$. As in the solution of a similar set of equations for the bounce point, there are special cases that cause the determinant of the coefficient matrix to be zero. This determinant is

$$\begin{vmatrix} UN_1 & UN_2 & UN_3 \\ UN_2 & -UN_1 & 0 \\ UN_3 & 0 & -UN_1 \end{vmatrix} = UN_1(UN^2 + UN_2^2 + UN_3^2) \tag{28}$$

The only possible case when this determinant would be zero occurs when $UN_1 = 0$. If this occurs, then $PN_1 = P_1 = x$ and a second system of equations can be determined such that

$$UN_2\ y + UN_3\ z = D_4$$

$$UN_3\ y - UN_2\ z = D_5 \tag{29}$$

where

$$D_4 = D_1 - UN_1P_1 = D_1$$

$$D_5 = UN_3P_2 - UN_2P_3 \tag{30}$$

The solution of this set yields PN_2 and PN_3. The determinant of the coefficient matrix of this last equation cannot be zero unless the surface unit normal vector is zero, which should not occur.

The portion of the time increment that is needed to travel from $\bar{P}$ to $\bar{PB}$ is given by

$$DTIME = DPB/VPP \tag{31}$$

This time segment is used to calculate the components of the tangential and normal velocities. The tangential velocity is determined from the equation of the line between the points $\bar{PN}$ and $\bar{PB}$:

$$\bar{PB} = \bar{PN} + \bar{v}(\Delta t) \tag{32}$$

The incidence tangential velocity components are expressed in the program as:

$$\begin{aligned} v_{t1} &= (PB_1 - PN_1)/DTIME \\ v_{t2} &= (PB_2 - PN_2)/DTIME \\ v_{t3} &= (PB_3 - PN_3)/DTIME \end{aligned} \tag{33}$$

In a similar way, the points $\bar{P}$ and $\bar{PN}$ are used to determine the normal component of the incidence velocity vector.

$$\begin{aligned} v_{n1} &= (PN_1 - P_1)/DTIME \\ v_{n2} &= (PN_2 - P_2)/DTIME \\ v_{n3} &= (PN_3 - P_3)/DTIME \end{aligned} \tag{34}$$

The subroutine RESTCO is then called to determine the normal and tangential restitution coefficients. These coefficients are dependent upon the magnitude of the incidence velocity and the incidence angle. ETA(1) is the normal restitution coefficient, and ETA(2) is the tangential restitution coefficient.

With these, the subroutine then determines the velocity components of the particle as it travels away from the point $\bar{PB}$.

$$\bar{VP} = ETA(1)\ \bar{V}_t + ETA(2)\ \bar{V}_n \tag{35}$$

Finally, the subroutine calculates the time elapsed, which is given by

$$T = T + DTIME \tag{36}$$

Method 2 is used when DPB < $\frac{1}{2}$ DPP. In this case, the particle travels over a very short distance along its trajectory between $\bar{P}$ and $\bar{\bar{P}}$ before it encounters the surface, including the case when the point P lies in the surface. Figure 8 illustrates the technique that is used.

The procedure is quite similar to Method 1, except that now, $\bar{\bar{P}}\bar{N}$ is the normal projection of the point PP onto the surface. This point $\bar{\bar{P}}\bar{N}$ is found by equations that are similar in form to those used in the previous method. The difference is that in the places where coordinates of $\bar{P}$ occurred in the previous equations, the coordinates of $\bar{\bar{P}}\bar{\bar{P}}$ now occur. Next the portion of the time increment that would be used to travel from $\bar{\bar{P}}\bar{B}$ to $\bar{\bar{P}}\bar{\bar{P}}$ is determined using

$$\text{DTIME} = H - \text{DPB}/\text{VPP} \tag{37}$$

The points $\bar{\bar{P}}\bar{\bar{P}}$, $\bar{\bar{P}}\bar{B}$, and $\bar{\bar{P}}\bar{N}$ are used to resolve the velocity into the tangential and normal components, which are given by

$$v_{ti} = (PN_i - PB_i)\text{DTIME} \quad i = 1, 2, 3 \tag{38}$$

$$v_{ni} = (PN_i - PP_i)\text{DTIME} \quad i = 1, 2, 3 \tag{39}$$

With the restitution coefficients found from RESTCO, the new velocities are determined as in the previous case. With the time correctly incremented to allow for the particle to travel to the surface, given by

$$T = T + \text{DPB}/\text{VPP} \tag{40}$$

The following pages contain a listing of the subroutine BOUNCE.

```
      SUBROUTINE BOUNCE(A,B,C,P,PP,H,T,ETA,NFIX,PB,VP)
      DIMENSION A(3),B(3),C(3),P(3),PP(3),V(3),AB(3),AC(3),G(3,3),D(5),
     1  PB(3)
      DIMENSION GS(3,3),VP(3),UN(3) ,PN(3)
      DIMENSION ETA(2)
      DIMENSION PNP(3),VN(4),VT(4)
C
      NFIX=1
      DO 10 I=1,3
      V(I)=(PP(I)-P(I))/H
      AB(I)=B(I)-A(I)
   10 AC(I)=C(I)-A(I)
      VPP=SQRT(V(1)**2+V(2)**2+V(3)**2)
C  DETERMINE UNIT NORMAL TO SURFACE
C
      UN(1)=AB(2)*AC(3)-AB(3)*AC(2)
      UN(2)=AB(3)*AC(1)-AB(1)*AC(3)
      UN(3)=AB(1)*AC(2)-AB(2)*AC(1)
      CMAG= SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
      IF( ABS(CMAG).GT.1.0E-12) GO TO 20
      NFIX=0
      WRITE(6,1000)
 1000 FORMAT(47H BOUNCE HAS ZERO UNIT VECTOR DESCRIBING SURFACE)
      RETURN
   20 DO 30 I=1,3
   30 UN(I)=UN(I)/CMAG
C
C  DETERMINE THE INTERSECTION POINT, PB, OF THE PLANE AND TRAJECTORY.
C
      DETA=UN(1)*V(1)**2+UN(2)*V(1)*V(2)+UN(3)*V(1)*V(3)
      D(1)=UN(1)*A(1)+UN(2)*A(2)+UN(3)*A(3)
      D(2)=V(2)*P(1)-V(1)*P(2)
      D(3)=V(3)*P(1)-V(1)*P(3)
      DO 40 J=1,3
   40 G(1,J)=UN(J)
      G(2,1)= V(2)
      G(2,2)=-V(1)
      G(2,3)=0.0
      G(3,1)= V(3)
      G(3,2)=0.0
      G(3,3)=-V(1)
C
C  IF DETERMINANT EQUALS ZERO, GO TO 90
C
      IF( ABS(DETA).LE.1.0E-12) GO TO 90
      DO 70 K=1,3
      DO 50 I=1,3
      DO 50 J=1,3
   50 GS(I,J)=G(I,J)
      DO 60 I=1,3
   60 GS(I,K)= D(I)
      PB(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
```

```
     2 -GS(3,3)*GS(2,1)*GS(1,2)
   70 PP(K)=PB(K)/DETA
      GO TO 100
C
C  IF DETERMINANT EQUALS ZERO, POINT P IS ON SURFACE, P EQUALS PB
C
   80 PB(1)=P(1)
      D(4)=D(1)-UN(1)*P(1)
      D(5)=V(3)*P(2)-V(2)*P(3)
      DETA=-UN(2)*V(2)-UN(3)*V(3)
      IF( ABS(DETA).LT.1.0E-12) GO TO 85
      PB(2)=(-D(4)*V(2)-D(5)*UN(3))/DETA
      PB(3)=(UN(2)*D(5)-V(3)*D(4))/DETA
      GO TO 100
   85 IF( ABS(V(3)).GT.1.0E-12) GO TO 90
      PB(2)=P(2)
      PB(3)=A(3)
      GO TO 100
   90 PB(2)=A(2)
      PB(3)=P(3)
  100 CONTINUE
      DPP= SQRT((PP(1)-P(1))**2+(PP(2)-P(2))**2+(PP(3)-P(3))**2)
      DPB= SQRT((PB(1)-P(1))**2+(PB(2)-P(2))**2+(PB(3)-P(3))**2)
      DPPB=SQRT((PP(1)-PB(1))**2+(PP(2)-PB(2))**2+(PP(3)-PB(3))**2)
      IF((DPPB.LT.DPP).AND.(DPB.LT.DPP)) GO TO 103
      NFIX=2
      RETURN
  103 CONTINUE
      IF(DPB.LT.(0.5*DPP)) GO TO 180
C
C  DETERMINE THE INTERSECTION POINT, PN, OF THE SURFACE NORMAL THRU P
C  IF DETERMINANT EQUALS ZERO, GO TO 140
C
      DETA=UN(1)**3+UN(1)*UN(2)**2+UN(1)*UN(3)**2
      IF( ABS(DETA).LE.1.0E-12) GO TO 140
      D(2)=P(1)*UN(2)-P(2)*UN(1)
      D(3)=P(1)*UN(3)-P(3)*UN(1)
      G(2,1)= UN(2)
      G(2,2)=-UN(1)
      G(2,3)=0.0
      G(3,1)= UN(3)
      G(3,2)=0.0
      G(3,3)=-UN(1)
      DO 130 K=1,3
      DO 110 I=1,3
      DO 110 J=1,3
  110 GS(I,J)=G(I,J)
      DO 120 I=1,3
  120 GS(I,K)=D(I)
      PN(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
     2 -GS(3,3)*GS(2,1)*GS(1,2)
  130 PN(K)=PN(K)/DETA
```

```
      GO TO 160
  140 PN(1)=P(1)
      D(4)=D(1)
      D(5)=UN(3)*P(2)-UN(2)*P(3)
      DETA=-UN(2)**2-UN(3)**2
      PN(2)=(-D(4)*UN(2)-D(5)*UN(3))/DETA
      PN(3)=(UN(2)*D(5)-UN(3)*D(4))/DETA
  160 CONTINUE
C
C  DETERMINE PORTION OF TIME SEGMENT USED TO TRAVEL FROM P TO PB.
C
      DTIME=DPB/VPP
      IF(DTIME.LT.H) GO TO 163
      WRITE(6,1010)
 1010 FORMAT(24H DTIME IS GREATER THAN H)
      NOHIT=0
      RETURN
C
C  EXTEND LINE PN-PB THE PROPER DISTANCE TO FIND PNP.
C  THEN EXTEND A LINE NORMAL TO THE SURFACE FROM PNP TO GET THE POINT
C  AFTER BOUNCE, PP.
C  FIND VELOCITY COORDINATES BASED ON PP, PB AND TIME REMAINING IN
C  SEGMENT.
C
  163 DO 165 I=1,3
      VT(I)=(PB(I)-PN(I))/DTIME
  165 VN(I)=(PN(I)-P(I))/DTIME
      VT(4)= SQRT(VT(1)**2+VT(2)**2+VT(3)**2)
      VN(4)= SQRT(VN(1)**2+VN(2)**2+VN(3)**2)
      CALL RESTCO(VN(4),VT(4),ETA)
      DO 170 I=1,3
      PNP(I)=PB(I)+ETA(2)*VT(I)*(H-DTIME)
      PP(I)=PNP(I)-ETA(1)*VN(I)*(H-DTIME)
  170 VP(I)=(PP(I)-PB(I))/(H-DTIME)
      T=T+H
      RETURN
C
C  IF POINT P LIES ON SURFACE, USE POINT PP TO DETERMINE AFTER
C  BOUNCE STATE.
C
  180 CONTINUE
      DETA=UN(1)**3+UN(1)*UN(2)**2+UN(1)*UN(3)**2
      IF( ABS(DETA).LE.1.0E-12) GO TO 220
      D(2)=PP(1)*UN(2)-PP(2)*UN(1)
      D(3)=PP(1)*UN(3)-PP(3)*UN(1)
      G(2,1)= UN(2)
      G(2,2)=-UN(1)
      G(2,3)=0.0
      G(3,1)=UN(3)
      G(3,2)=0.0
      G(3,3)=-UN(1)
      DO 210 K=1,3
      DO 190 I=1,3
```

```
      DO 190 J=1,3
  190 GS(I,J)=C(I,J)
      DO 200 I=1,3
  200 GS(I,K)=D(I)
      PN(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
     2 -GS(3,3)*GS(2,1)*GS(1,2)
  210 PN(K)=PN(K)/DETA
      GO TO 240
  220 PN(1)=PP(1)
      D(4)=D(1)
      D(5)=UN(3)*PP(2)-UN(2)*PP(3)
      DETA=-UN(2)*UN(2)-UN(3)*UN(3)
      PN(2)=(-D(4)*UN(2)-D(5)*UN(3))/DETA
      PN(3)=(UN(2)*D(5)-UN(3)*D(4))/DETA
  240 CONTINUE
C
C  DETERMINE PORTION OF TIME SEGMENT REMAINING FOR TRAVEL FROM PB TO PP.
C
      DTIME=H-DPP/VPP
      IF(DTIME.GT.1.0E-12) GO TO 245
      WRITE(6,1020)
 1020 FORMAT(21H DTIME LESS THAN ZERO)
      NEXT=0
      RETURN
C
C  DETERMINE THE PROPER DISTANCE ALONG PN-PB TO FIND PNP.
C  THEN EXTENT A LINE NORMAL TO THE SURFACE FROM PNP TO GET THE POINT
C  AFTER BOUNCE, PP.
C  FIND VELOCITY COORDINATES BASED ON PP, PB, AND THE TIME H.
C
  245 DO 250 I=1,3
      VT(I)=(PN(I)-PB(I))/DTIME
  250 VN(I)=(PP(I)-PN(I))/DTIME
      VT(4)= SQRT(VT(1)**2+VT(2)**2+VT(3)**2)
      VN(4)= SQRT(VN(1)**2+VN(2)**2+VN(3)**2)
      CALL RESTCO(VN(4),VT(4),ETA)
      DO 260 I=1,3
      PNP(I)=PB(I)+ETA(2)*VT(I)*DTIME
      PP(I)=PNP(I)-ETA(1)*VN(I)*DTIME
  260 VP(I)=(PP(I)-PB(I))/DTIME
      T=T+H
      RETURN
      END
```

Subroutine RESTCO

It was necessary to develop expressions for the normal and tangential restitution coefficients to describe the momentum loss experienced by a particle when it bounces off a surface. Initially, constant values of restitution coefficients were used. As a result, the particles that struck the surfaces many times, lost momentum with each bounce and eventually came to rest against the surface. This is unrealistic to occur, and therefore the available data on restitution coefficients was reviewed and the information obtained used to develop a set of empirical equations that would describe the restitution coefficient for all incidence velocities and all incidence angles.

Data given by Grant (6) and Ball (7) indicate that the restitution coefficients were functions of both the incidence angle and the particle incidence velocity. The data given by Ball (7) is perhaps more realistic because it applies to Titanium and Stainless Steel alloys which are more typical of blade materials that are used in radial inflow turbines. However, this data is not as extensive as the data given by Grant (6) for an Aluminum alloy. Because the more extensive data gave a better description of the variations that could be expected, the aluminum alloy data was used.

The emperical expressions arrived at for the normal and tangential restitution coefficients are

$$\eta_N(\beta,V) = 1.0 - \psi_1(\beta)\ \psi_2(V)$$

$$\eta_T(\beta,V) = 1.0 - \phi_1(\beta)\ \phi_2(V) \qquad (41)$$

In this expression, η_N and η_T are the normal and tangential restitution coefficients and ψ_1, ψ_2, ϕ_1, ϕ_2 are empirical functions which go to zero as the argument goes to zero.

Figure 9 shows the data obtained by Grant (6) for the normal restitution coefficient variation as the impingement angle changes. An assumption was made that ψ_2 is one when the velocity is 76.2 meters/sec., and then a polynomial curve was fitted to the data

points. Table 1 gives the coefficients of this polynomial for the normal restitution coefficients. The variation of the normal restitution coefficient with the incidence velocity is shown in Figure 10. The experimental data points of Reference 6 are shown in the same Figure. It was assumed that the normal restitution coefficient increases to one as the incidence velocity decreases to zero. In order to develop the empirical equation for ψ_2, the value of ψ_1 at 45^o was used. A value of ψ_2 equal to one was taken for incidence velocity of 76.2 meters/sec. The following expression was developed to express ψ_2:

$$\psi_2 = 0.65(1.0 - e^{-V/24.5}) \tag{42}$$

Equation (42) and the polynomial expression for the variations due to the incidence angle were combined into a complete empirical formula for the normal restitution coefficient. Figure 11 illustrates the variation in the restitution coefficient with the incidence velocity and the incidence angles, according to the emperical equation.

In a similar way, the data given in Reference 6 for the tangential restitution coefficients at an incidence velocity of 76.2 meters/sec., was fitted with a polynomial expression as shown in Figure 12. In addition to the experiment 1 data points that were used, it was assumed that the tangential restitution coefficient was equal to one for incidence angles of 0^o and 90^o. The last two values were used, with the experimental data and these points were also used in the evaluation of the polynomial coefficients which are given in Table 2. Data on the variation of the tangential restitution coefficient with incidence velocity is indicated in Figure 13. It was assumed that the restitution coefficient goes to one as the tangential velocity goes to zero, this and the two points of the experimental data showed linear variation. Figure 14 shows the family of curves of the tangential restitution coefficient versus incidence angles for different incidence velocities.

Because of the linear variation in restitution coefficient with incidence velocities, very low restitution coefficients are calculated for very high incidence velocities. A lower limit of 0.1 was placed on the allowable value of the restitution coefficient as indicated in Figure 14.

The next page contains a listing of the subroutine RESTCO, which receives the normal and tangential velocity components from the calling program and returns the proper values of the restitution coefficients.

```
      SUBROUTINE RESTCO(VN,VT,ETA)
         DIMENSION ETA(2),A(10),B(10)
      DATA A/1.8190,5.1171,-49.2529,131.03528,-174.4249,117.8723,
     1  -24.47885,-16.7297,11.2300,-1.979996/
      DATA B/5.7215,-41.8808,178.1685,-424.3882,572.7631,-406.6625,
     1  87.1428,70.6511,-50.4982,9.6767/

C  DATA IN THIS SUBROUTINE CORRESPONDS TO VELOCITIES IN FT/SEC.
C  MATERIAL TYPICAL OF ALUMINUM AND SILICON PARTICLES.
C  ETA(1) IS THE NORMAL RESTITUTION COEFFICIENT.
C  ETA(2) IS THE TANGENTIAL RESTITUTION COEFFICIENT.
C
      BETA=ATAN2(VN,VT)
      V=SQRT(VN**2+VT**2)
      PHIONE=V/250.00
      PHITWO= A(1)*BETA+A(2)*BETA**2+A(3)*BETA**3+A(4)*BETA**4
     1  +A(5)*BETA**5+A(6)*BETA**6+A(7)*BETA**7+A(8)*BETA**8
     2  +A(9)*BETA**9+A(10)*BETA**10
      PHI=PHIONE*PHITWO
      IF(PHI.GT.0.90) PHI=0.90
      ETA(2)=1.000-PHI
      PSIONE=1.0000-EXP(-V/36.000)
      PSITWO= B(1)*BETA+B(2)*BETA**2+B(3)*BETA**3+B(4)*BETA**4
     1  +B(5)*BETA**5+B(6)*BETA**6+B(7)*BETA**7+B(8)*BETA**8
     1  +B(9)*BETA**9+B(10)*BETA**10
      PSI=PSIONE*PSITWO
      IF(PSI.GT.0.900) PSI=0.900
      ETA(1)=1.000-PSI
      RETURN
      END
```

Subroutine RNUMBR

Another subroutine that is used throughout this set of programs is the subroutine RNUMBR. This subroutine uses the Reynolds number determined by the calling program to find the drag coefficient for a sphere. The equations which are used to describe the drag coefficient are:

$$C_D = 4.5 + \frac{24}{Re} \qquad Re < 1.0$$

$$C_D = 28.5 - 24.0\ (\text{Log Re}) + 9.0682\ (\text{Log Re})^2$$

$$- 1.7713\ (\text{Log Re})^3 + 0.1718\ (\text{Log Re})^4 - 0.0065\ (\text{Log Re})^5$$

$$1.0 < Re < 3000$$

$$C_D = 0.4 \qquad 3000 < Re < 2.5 \times 10^{-5} \qquad (43)$$

This subroutine includes a factor DGFC, which can be used to determine the drag coefficient for non-spherical particles. A listing of this subroutine follows.

Program Listing RNUMBR

```
      SUBROUTINE RNUMBR(RENOLD,DCFC,CD)
      IF( ABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DCFC*0.4
      RETURN
   26 CD=DCFC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=ALOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1  -0.0065*ARE**5)*DCFC
      RETURN
      END
```

Subroutines ALOCAT, BLOCAT, and DLOCAT

This series of subroutines are used to determine the location of the particle within a grid pattern so that the gas velocities and flow properties can be determined. Although each subroutine does approximately the same thing, it was found that each flow geometry required slightly different forms of the subroutine. Most of the geometries used in this study are such that the gas flow is described at nodes along the quasi-orthogonals of the flow. Generally, the indexes that locate the nodes are in increasing order from inlet to exit. All of these subroutines check each orthogonal to find the first orthogonal beyond the particle location. The subroutine then sweeps from hub to shroud, or from blade to blade along the stream surface until it determines the index of the streamline just beyond the particle location, then specifies the four grid points surrounding the particle. The subroutines check if the particle is out of bounds. If the particle is in bounds, then NOWT = 0, otherwise the code work NOWT is set equal to a number that depends on the boundary. These subroutines are included with the individual program listings.

Subroutine POLATE

This subroutine interpolates linearly the properties of the gas flow at the location of the particle within a grid, using the known flow properties at the four grid points that surround the particle. Figure 15 illustrates the grid points surrounding the particles that determined by the locating subroutines. The distances from the orthogonals to the particle location, D_1 and D_2, are determined first. Next the program calculates the distances from AA and AB to the particle location. All of these distances are used to interpolate the value of any flow property A. The same method is used to determine the r, z coordinates of the properties. The subroutine returns this property to the main program as AP.

There are minor differences in the subroutines that are listed with each of the programs. These minor differences occur because of the significant differences in the geometries between different flow fields. For example, the STATOR program uses an r-θ grid while the ROTOR program uses an r-z grid. However, the subroutines all follow a procedure similar to the one outlined here.

Function RUNGE

This function uses a 4th order Runge-Kutta technique to integrate a system of simultaneous first order ordinary differential equations over one time increment. A more complete description of the subroutine and the input and output quantities can be found in Reference 5. A listing of the subroutine is included in that reference.

PROGRAM DESCRIPTIONS

The following sections will describe the programs, their input and output and other information that will aid in understanding their use. A complete listing of the program is included with a sample set of data to demonstrate the program output.

PARDIM - Particles in a Vortex

This program integrates the equations of motion to determine the trajectory of the particle in inward flowing free vortices, or whirling flows that have no radial components but do have axial components. The force of gravity acting on the particle can be included in cases where gravity must be considered. The solution is essentially a three dimensional one, but instead of allowing particles to bounce, the integration stops when the particle passes boundaries of the fluid flow. The cylindrical coordinates, r, θ and z, are used as indicated in Figure 16.

Method

Figure 17 is a flow diagram of the program PARDIM. The subroutine, VORTEX is used to provide the three components of the gas velocity and the necessary gas properties within the boundaries of the flow field. The function RUNGE is used to integrate the equations of motion. The general iteration technique that is used to calculate the average values of the gas properties at the particle location, and the method used to integrate the equations of motion of the particle have been explained before.

The first call of subroutine VORTEX allows the general characteristics of the gas flow to be introduced into the subroutine. Subsequent calls return the velocity components, static gas temperature and density that are the solution of the free vortex flow at the radial location of the particle. The equations that govern the gas flow field are; conservation of momentum, $\lambda = rV_u$ = constant, the conservation of mass, $\frac{w}{2\pi b} = \rho V_r r$ = constant, and isentropic flow relations.

The solution for the mass flow in the subroutine VORTEX uses an iteration technique that will sometimes fail to converge when the velocities become large. If this occurs, the subroutine prints "VORTEX FAILED TO CONVERGE AT LOCATION R = ," and then sets a parameter that causes the main program to proceed to the next data set. A second message can sometimes be printed by the same subroutine if the flow of gas in a compressible free vortex has a supersonic solution. The message "CHOKED FLOW" will be printed in such case.

Input

There are two sets of input. The first set specifies the nature of the gas flow and consists of 4 cards at the front of the input data. This program is written so than any consistent system of units may be used.

The first four data cards take the following form.

DANGLE, TMAX, RMIN, RMAX, ANGMAX, ZMAX	(6F10.4)
VISREF, TREF, TSUT, GAM, RGAS, DGFC	(E20.5, 5F10.4)
RIN, VRIN, VUIN, VZIN, TT, RHOT	(5F10.4, E10.4)
ALPHA, BETA, GRAVITY	(3F10.4)

These variables are defined later.

The second set of data cards specifies the particle size and density and the initial position and velocity of the particle. Each particle can be described by four input data cards which take the following form.

TITLE	(18A4)
(YR(I), I = 1,6)	(6F10.6)
RHOP, DIAP	(F10.6, E10.4)
NSTEP, H	(I10, E10.4)

For studies that are done with multiple particles, additional second sets of input data cards may be stacked together. When the program completes one set it goes to the next set automatically. An explanation of the input variables of both sets follows.

DANGLE - Data is printed every DANGLE degrees.

TMAX - The program truncates the solution when time exceeds TMAX seconds and goes to the next particle.

RMIN - The program stops and goes to the next data set when the particle radial location is smaller than RMIN. (Length units).

RMAX - The program stops and goes to the next data set when the particle radial location is larger than RMAX. (Length units).

ANGMAX - The program stops and goes to the next data set when the particle angular position is greater than ANGMAX in degrees.

ZMAX - The program stops and goes to the next data set when the particle axial position is greater than ZMAX. (Length units).

VISREF - Reference viscosity corresponding to TREF. Used in Sutherland's Law. (Mass/Length x Time).

TREF - Reference temperature corresponding to VISREF. Used in Sutherland's Law. (Absolute degrees).

TSUT - Constant used in Sutherland's Law. (198.6^{o}R or 110^{o}K).

GAM - Ratio of specific heats.

RGAS - Gas constant. (Length^2/Time^2 Deg. Abs.).

DGFC - Drag Factor - The spherical drag coefficient based on Reynolds Number is multiplied by DGFC. Except in very special cases, this should be 1.0.

RIN - Radius at inlet of the gas stream. (Length).

VRIN, VUIN, VZIN - Radial, tangential, and axial components of the gas velocity at RIN. In an inward flowing vortex, the radial component should be negative. (Length/Time).

TT - Stagnation inlet temperature. (Degrees Abs.).

RHOT - Stagnation inlet density. (Mass/Length^3).

ALPHA - Angle of Z-axis with respect to the horizontal. (Degrees). See Figure 18.

BETA - Angle of the turbine verticle axis with respect to the gravitational vertical. (Degrees). See Figure 18.

GRAVTY - Acceleration of Gravity. (Length/Time^2).
TITLE - The first card is reproduced at the top of the first page of output for each particle. Any statement in columns 2 to 72 will be reproduced.
YR(1) - Initial particle radial position. (Length).
YR(2) - Initial particle radial velocity. (Length/Time). For inward moving particle, the radial component is negative.
YR(3) - Initial particle angular position, θ. (Degrees).
YR(4) - Initial particle angular velocity, $\dot{\theta}$. (Radians/Sec).
YR(5) - Initial particle axial position. (Length).
YR(6) - Initial particle axial velocity. (Length/Time).
RHOP - Particle density. (Mass/Length^3).
DIA - Particle diameter. (Length).
NSTEP - An integer that is not used in this program.
H - Integration time increment. (Time).

Output

The first part of the output is an echo check of the first set of data cards which are used to describe the gas flow field. Such data checks are useful in correcting key punch mistakes on the input cards. After initializing these variables, the program calculates and prints the critical gas velocity. This critical velocity is determined based on the gas stagnation temperature. The following print out starts on the next page, and the printed output data for each particle starts at a new page. The first part of the printed output is an echo check of the input data as punched on the second set of data cards. After initializing the variables, the program calculates and prints several similarity parameters that are useful in relating particles that have similar trajectories. Next, the particle position and velocities at every DANGLE degrees are printed. The parameters appearing in the output, that were not defined before in the input data, are listed below.

DELTA - Characteristic length as given in Equation 7. (Length).
TAU - Time constant as given in Equation 8. (Time).
RECR - Reynolds number as given in Equation 9.
T - Time. (Time).
RENOLDS - Reynolds number for the particle at this point.
R/RIN - Normalized radial position of the particle.
STREAMLINE - Normalized radial coordinate of an incompressible flow streamline that would exist in a free vortex starting at RIN with the same initial velocity components given as input. Such a streamline will follow the equation $R/RIN = e^{(\theta \tan \alpha_1)}$ where α_1 = arctan (VRIN/VUIN) [8]. The subroutine VORTEX provides a compressible solution, thus STREAMLINE is only provided for comparison purposes.

When the solution is complete, the program writes all the trajectory information at the last point. Also printed are, M, the iteration counter and L, the time increment counter. If the air velocities fail to converge in 100 iterations, the program stops, prints the trajectory data from the last iteration and M = 101.

```
C   DIMENSIONAL SOLUTION AND PRINT OUT FOR PARTICLE TRAJECTORIES.
C   ANY CONSISTENT SYSTEM OF UNITS.
C
      DIMENSION VR(4),VU(4),YR(6),FR(6),YRS(6),VZ(4),TEMP(2),RHO(2)
      DIMENSION STATE(18)
      INTEGER RUNGE
      READ(5,1000)  DANGLE,TMAX,RMIN,RMAX,ANGMAX,ZMAX
      WRITE(6,2000) DANGLE,TMAX,RMIN,RMAX,ANGMAX,ZMAX
      READ(5,1100)  VISREF,TREF,TSUT,GAM,RGAS,DGEC
      WRITE(6,2100) VISREF,TREF,TSUT,GAM,RGAS,DGEC
      READ(5,1200) RIN,VRIN,VUIN,VZIN,TT,RHOT
      WRITE(6,2200)RIN,VRIN,VUIN,VZIN,TT,RHOT
      READ(5,1000)  ALPHA,BETA,GRAVTY
      WRITE(6,2510) ALPHA,BETA,GRAVTY
      IF(ABS(VUIN).LT.1.0E-10) VUIN=1.0E-10
      TALPH=VRIN/VUIN
      SALPH=SIN(ALPHA/57.29578)
      BETA=BETA/57.29578
      CVGR=1.0/(GAM-1.0)
      CPGR=GAM/(GAM-1.0)
      ABC=(GAM-1.0)/(GAM+1.0)
      VCR=SQRT(2.0*RGAS*GAM*TT/(GAM+1.0))
      WRITE(6,2400)VCR
      DO 205 NUMB=1,100
      READ(5,1400) (STATE(I),I=1,18)
      WRITE(6,2700)(STATE(I),I=1,18)
      READ(5,1300)  (YR(I),I=1,6)
      WRITE(6,2300) (YR(I),I=1,6)
      READ(5,1500) RHOP, DIA
      WRITE(6,2110) RHOP,DIA
      READ(5,1510) NSTEP,H
      WRITE(6,2120)NSTEP,H
C
C   CONVERT TO NON-DIMENSIONAL QUANTITIES AND INITIALIZE
C
      RHOCR=RHOT*(2.0/(GAM+1.0))**CVGR
      DELTA=RHOP*DIA/RHOCR/0.3
      TCR=TT*2.0/(GAM  +1.0)
      VISCR=VISREF*((TCR/TREF)**1.5)*(TREF+TSUT)/(TCR+TSUT)
      TAU=RHOP*DIA**2/18.0/VISCR
      RECR=DIA*RHOCR*VCR/VISCR/2.0
      WRITE(6,2900) DELTA,TAU,RECR
      WRITE(6,2410)
      TEMP(2)=TT
      RHO(2)=RHOT
      YR(3)=YR(3)/57.29577
      VR(1)=VRIN
      VU(1)=VUIN
      VZ(1)=VZIN
      N=6
      L=-1
      M=1
      T=0.0
```

```
      ANGLE=YR(3)
      TS=T
      DO 10 I=1,4
      VR(I)=VR(1)
      VZ(I)=VZ(1)
   10 VU(I)=VU(1)
      DO 20 I=1,N
   20 YRS(I)=YR(I)
      CALL VORTEX(L,VRIN,VUIN,VZIN,RIN,TEMP,RHO,GAM,RGAS)
      L=L+1
      VISTAR=VISREF*((TEMP(1)/TREF)**1.5)*(TREF+TSUT)/(TEMP(1)+TSUT)
C
C  INTEGRATE USING RUNGE-KUTTA METHOD
C
   25 VDIFF=SQRT((VR(2)-YR(2))**2+(VU(2)-YR(1)*YR(4))**2
     1  +(VZ(2)-YR(6))**2)
      RENOLD=RHO(1)*VDIFF*DIA/VISTAR
      CALL RNUMBR(RENOLD,1.0,CD)
      BCON=RHO(1)*CD/RHOP/DIA/1.166667
   30 IF(RUNGE(N,YR,FR,T,H).NE.1) GO TO 40
      FR(1) =       YR(2)
      FR(2) =       YR(1)*YR(4)**2+BCON*VDIFF*(VR(2)-YR(2))
     1  -GRAVTY*COS(YR(3)+BETA)
      FR(3) =       YR(4)
      FR(4) =       -2.0*YR(2)*YR(4)/YR(1)+BCON*VDIFF
     1              *(VU(2)-YR(1)*YR(4))/YR(1)
     2  +GRAVTY*SIN(YR(3)+BETA)
      FR(5)=        YR(6)
      FR(6)=        BCON*VDIFF*(VZ(2)-YR(6))
     1  -GRAVTY*SALPH
      GO TO 30
   40 CALL VORTEX(L,VR(4),VU(4),VZ(4),YR(1),TEMP,RHO,GAM,RGAS)
      IF(L.EQ.10000) GO TO 205
C
C  TEST AIR VELOCITY VALUES USED,  IF INCORRECT, RESET INTEGRATION
C  VALUES AND GO TO 25,  IF CORRECT, GO TO 50
C
      IF((ABS(VR(4)-VR(3)).LT.1.0E-4).AND.(ABS(VU(4)-VU(3)).LT.1.0E-4)
     1  .AND.(ABS(VZ(4)-VZ(3)).LT.1.0E-4)) GO TO 50
      VR(2)=(VR(4)+VR(1))/2.0
      VU(2)=(VU(4)+VU(1))/2.0
      VZ(2)=(VZ(4)+VZ(1))/2.0
      VR(3)=VR(4)
      VU(3)=VU(4)
      VZ(3)=VZ(4)
      T=TS
      DO 45 I=1,N
   45 YR(I)=YRS(I)
      IF(M.GT.100) GO TO 200
      M=M+1
      GO TO 25
C
C  COMPLETE INTEGRATION STEP.  IF REQ'D, WRITE OUTPUT.  GO TO 30
```

```
C
   50 M=1
      L=L+1
      TS=T
      THETA=YR(3)*57.29578
      DO 60 I=1,M
   60 YRS(I)=YR(I)
      VR(2)=1.5*VR(4)-0.5*VR(1)
      VR(3)=2.0*VR(4)-VR(1)
      VR(1)=VR(4)
      VR(4)=VR(3)
      VU(2)=1.5*VU(4)-0.5*VU(1)
      VU(3)=2.0*VU(4)-VU(1)
      VU(1)=VU(4)
      VU(4)=VU(3)
      VZ(2)=1.5*VZ(4)-0.5*VZ(1)
      VZ(3)=2.0*VZ(4)-VZ(1)
      VZ(1)=VZ(4)
      VZ(4)=VZ(3)
      VISTAR=VISREF*((TEMP(1)/TREF)**1.5)*(TREF+TSUT)/(TEMP(1)+TSUT)
   80 IF((T.GT.TMAX).OR.(YR(1).LT.RMIN).OR.(YR(1).GT.RMAX)
     1  .OR.(THETA.GT.ANGMAX).OR.(YR(5).GT.ZMAX)) GO TO 200
      IF((THETA-ANGLE).LE.0.0) GO TO 30
      RTD=YR(4)
      SLINE=EXP(YR(3)*TALPH)
      RORIN=YR(1)/RIN
      WRITE(6,2500) T,YR(1),YR(2),THETA,RTD,YR(5),YR(6),RENOLD,RORIN,
     1  SLINE
      ANGLE=ANGLE+DANGLE
      GO TO 25
  200 CONTINUE
      RTD=YR(4)
      SLINE=EXP(YR(3)*TALPH)
      RORIN=YR(1)/RIN
      WRITE(6,2600) T,YR(1),YR(2),THETA,RTD,YR(5),YR(6),RENOLD,RORIN,
     1  SLINE
      WRITE(6,2800) M,L
  205 CONTINUE
 1000 FORMAT(7F10.4)
 1100 FORMAT(F20.5,5F10.4)
 1200 FORMAT(5F10.4,F10.4)
 1300 FORMAT(6F10.6)
 1400 FORMAT(18A4)
 1500 FORMAT(F10.6,F10.4)
 1510 FORMAT(I10,E10.3)
 2000 FORMAT(1H0,7X,7HD-ANGLE,10X,5HT-MAX,10X,5HR-MIN,10X,5HR-MAX,6X,
     1  9HMAX-ANGLE,10X,5HZ-MAX,/,(6F15.4))
 2100 FORMAT(1H0,5X,14HREF. VISCOSITY,6X,9HREF. TEMP,2X,13HSUTHERLANDS T
     1  , 10X,5HGAMMA,6X,9HGAS CONST,4X,11HDRAG FACTOR,/,(F20.5,5F15.4))
 2110 FORMAT(1H0,10X,4HRHOP,12X,3HDIA,/,(F15.6,F15.4))
 2120 FORMAT(1H0, 9X,5HNSTEP,14X,1HH,/,(I15,F15.3))
 2200 FORMAT(1H0,11X,3HRIN,11X,4HVRIN,11X,4HVUIN,11X,4HVZIN,13X,2HTT,
     1  11X,4HRHOT,/,(5F15.4,F15.4))
```

```
 2300 FORMAT(1HO, 9X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,5HYR(4),10X,
     1   5HYR(5),10X,5HYR(6),/,(6F15.6))
 2400 FORMAT(1HO,19HCRITICAL VELOCITY =,F10.4)
 2410 FORMAT(1HO,8X,1HT,7X,5HYR(1),7X,5HYR(2),7X,5HYR(3),7X,5HYR(4),
     1   7X,5HYR(5),7X,5HYR(6),5X,7HRENOLDS,7X,5HR/RIN,2X,10HSTREAMLINE)
 2500 FORMAT(1H ,F10.2,3F12.5,F12.2,2F12.5,F12.3,2F12.5)
 2510 FORMAT(1HO,10X,5HALPHA,11X,4HBETA,2X,13HACC. OF GRAV.,/,(3F15.4))
 2600 FORMAT(1HO,F7.3,F12.4,2F13.4,3F12.4,F13.4,F10.4,F12.4)
 2700 FORMAT(1H1,/,18A4)
 2800 FORMAT(1H ,5X,3HM =,I5,5X,3HL =,I10)
 2900 FORMAT(32HOSIMILARITY PARAMETERS.  DELTA =,F10.4,5X,5HTAU =,F12.4,
     1   5X,6HRECP =,F12.4)
      STOP
      END
```

The function routine RUNGE has been removed from the published form of this report to protect the copyright of the authors of Reference 5.

```
      SUBROUTINE RNUMBR(RENOLD,DGEC,CD)
      IF( ABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DGEC*0.4
      RETURN
   26 CD=DGEC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=ALOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1   -0.0065*ARE**5)*DGEC
      RETURN
      END
```

```
      SUBROUTINE VORTEX(L,VR,VU,VZ,POS,T,RHO,GAM,RGAS)
      DIMENSION T(2),RHO(2)
      M=0
      IF(L.GE.0) GO TO 10
      N=0
      CVOR=1.0/(GAM-1.0)
      ABC=(GAM-1.0)/(GAM+1.0)
      VCR=SQRT(2.0*RGAS*GAM*T(2)/(GAM+1.0))
      TOTT=1.0-ABC*((VU**2+VR**2+VZ**2)/(VCR**2))
      T(1)=T(2)*TOTT
      RHO(1)=RHO(2)*(TOTT**CVOR)
      CU=VU*POS
      CR=-VR*POS*RHO(1)/RHO(2)
      RETURN
   10 IF(M.GT.100) GO TO 20
      VRP=VR
      VUP=VU
      TOTT=1.0-ABC*((VU**2+VR**2+VZ**2)/(VCR**2))
      IF(TOTT.LE.0.0) GO TO 15
      VU=CU/POS
      VR=-CR*((1.0/TOTT)**CVOR)/POS
      M=M+1
      IF((ABS(VRP-VR).GT.1.0E-6).AND.(ABS(VUP-VU).GT.1.0E-6)) GO TO 10
      T(1)=T(2)*TOTT
      RHO(1)=RHO(2)*(TOTT**CVOR)
      AMACH=SQRT((VU**2+VR**2+VZ**2)/(GAM*RGAS*T(1)))
      IF(AMACH.GT.1.0) GO TO 30
      GO TO 40
   15 WRITE(6,1200) POS
 1200 FORMAT(1H0,41HVORTEX FAILED TO CONVERGE AT LOCATION R =,F10.4)
      L=10000
      RETURN
   20 WRITE(6,1000) M,VR,VRP,VU,VUP,TOTT
      GO TO 40
   30 IF(N.EQ.0) WRITE(6,1100)
      N=1
   40 CONTINUE
 1000 FORMAT(I10,5E15.8)
 1100 FORMAT(1H0,11HCHOKED FLOW)
      RETURN
      END
```

Example

The example presented here uses the ft., lbm., second system of units. The vortex in this case has an inlet radius of 0.2742 ft. and an exit of 0.2467 ft. The axial span of the vortex lies between z = 0 and z = 0.0264 ft. A cold gas equivalent flow is considered and therefore, the inlet stagnation conditions are standard sea level conditions. Gravity is neglected in this example.

The specific gravity of the particle studied is about 3, its diameter is 236 microns, its initial velocity is equal to one half of the gas velocity and its initial position is slightly below the inlet radius. After 27.3°, the particle leaves the flow field at the inlet radius.

The following pages contain a computer code sheet with the data arranged in the proper columns and the output for this example.

PARDIM Example Case for NASA Report

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

1.0	0.0032	0.2467	0.2742	360.0	0.0264		CARD SET 1
	0.106E-4	492.0	198.2	1.4	1715.48	1.0	
0.2741	-184.0	780.0	0.0	518.7	0.764E-1		
0.0	0.0	0.0					
EXAMPLE CASE FØR NASA REPØRT V-PART/V-GAS = 50%							
0.2741	-92.0	0.0	1422.84	0.001	0.0		CARD SET 2
187.2	0.7761E-3						
10	0.1E-4						
Additional particle trajectories require a Card Set 2 arrangement here.							
These sets are stacked one after the other.							CARD SET 2

P-ANGLE	T-MAX	R-MIN	R-MAX	MAX-ANGLE	Z-MAX
1.0000	0.0032	0.2467	0.2742	360.0000	0.0264

REF. VISCOSITY	REF. TEMP	SUTHERLANDS T	GAMMA	GAS CONST	DRAG FACTOR
0.10600E-04	492.0000	198.2000	1.4000	1715.4800	1.0000

RIN	VRIN	VTIN	VZIN	TT	RHCT
0.2741	-184.0000	780.0000	0.0	518.7000	0.7640E-01

ALPHA	BETA	ACC. OF GRAV.
0.0	0.0	0.0

CRITICAL VELOCITY = 1018.8826

EXAMPLE CASE FOR NASA REPORT V-PART/V-GAS = 50%

YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)
0.274100	-92.000000	0.0	1422.839844	0.001000	0.0

RHOP	DIA
197.199997	0.7761E-03

NSTEP	H
10	0.100E-04

SIMILARITY PARAMETERS. DELTA = 5.5551 TAU = 0.6555E 00 RECR = 0.2004E 04

T	YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)	REYNOLDS	R/RIA	STREAMLINE
0.10E-04	0.27321	-86.47282	0.81814	1432.94	0.00100	0.0	0.179E 04	0.99674	0.99664
0.20E-04	0.27237	-80.89003	1.64193	1442.55	0.00100	0.0	0.180E 04	0.99369	0.99326
0.30E-04	0.27159	-75.25435	2.47109	1451.67	0.00100	0.0	0.181E 04	0.99084	0.98988
0.40E-04	0.27087	-69.56868	3.30532	1460.25	0.00100	0.0	0.181E 04	0.98820	0.98648
0.50E-04	0.27023	-63.83595	4.14431	1468.29	0.00100	0.0	0.182E 04	0.98577	0.98308
0.60E-04	0.26964	-58.04249	4.98529	1482.65	0.00100	0.0	0.184E 04	0.98153	0.97626
0.80E-04	0.26854	-46.38884	6.66661	1488.94	0.00100	0.0	0.185E 04	0.97973	0.97285
0.90E-04	0.26811	-40.50673	7.54136	1494.62	0.00100	0.0	0.186E 04	0.97815	0.96943
0.10E-03	0.26773	-34.56330	8.39519	1499.67	0.00100	0.0	0.187E 04	0.97678	0.96601
0.11E-03	0.26742	-28.63979	9.25973	1504.37	0.00100	0.0	0.188E 04	0.97562	0.96259
0.12E-03	0.26716	-22.67404	10.12260	1507.83	0.00100	0.0	0.189E 04	0.97469	0.95918
0.14E-03	0.26683	-10.69171	11.85396	1513.35	0.00100	0.0	0.191E 04	0.97347	0.95237
0.15E-03	0.26675	-4.68316	12.72148	1515.10	0.00100	0.0	0.192E 04	0.97319	0.94897
0.16E-03	0.26673	1.33156	13.58991	1516.17	0.00100	0.0	0.194E 04	0.97313	0.94558
0.17E-03	0.26678	7.34834	14.45875	1516.57	0.00100	0.0	0.195E 04	0.97328	0.94221
0.18E-03	0.26689	13.36304	15.32743	1516.29	0.00100	0.0	0.196E 04	0.97366	0.93884
0.19E-03	0.26704	19.37157	16.19615	1515.32	0.00100	0.0	0.197E 04	0.97426	0.93549
0.21E-03	0.26727	25.36983	17.06392	1513.68	0.00100	0.0	0.198E 04	0.97508	0.93216
0.22E-03	0.26750	37.31931	18.79570	1508.40	0.00100	0.0	0.200E 04	0.97736	0.92553
0.23E-03	0.26783	43.26256	19.65894	1504.78	0.00100	0.0	0.201E 04	0.97883	0.92225
0.24E-03	0.26826	49.17960	20.51993	1500.51	0.00100	0.0	0.202E 04	0.98052	0.91899
0.25E-03	0.26928	55.06662	21.37928	1495.61	0.00100	0.0	0.203E 04	0.98242	0.91574
0.26E-03	0.26986	60.91992	22.23366	1490.10	0.00100	0.0	0.204E 04	0.98454	0.91252
0.27E-03	0.27050	66.73585	23.08569	1483.98	0.00100	0.0	0.205E 04	0.98687	0.90933
0.28E-03	0.27195	78.24197	24.77841	1470.03	0.00100	0.0	0.207E 04	0.99216	0.90301
0.30E-03	0.27274	83.92545	25.61844	1462.18	0.00100	0.0	0.208E 04	0.99511	0.89990
0.31E-03	0.27363	89.55849	26.45383	1453.82	0.00100	0.0	0.209E 04	0.99828	0.89661
0.300	0.2746	95.1980	27.2843	1444.9573	0.0010	0.0	2056.6499	1.0016	0.8937

M = 1 L = 32

SCRL2D - Two-dimensional Scroll

This program integrates the equations of motion to determine the trajectory of a particle in a scroll. The force of gravity acting on the particle can be included in the cases where this is necessary. The solution is two dimensional, but the output has been written to include a possible extension in the future to a three-dimensional solution. The particle is allowed to bounce off the scroll wall, and the solution stops when the particle enters the nozzle region of the flow. The cylindrical coordinate system r, θ and z is used as indicated in Figure 16.

Method

Figure 19 is a flow diagram of this program. Unlike the previous cases, there is no need to iterate for the average gas flow properties at the location of the particle. The gas flow solution is assumed to be uniform throughout the scroll and, therefore, the properties of the gas do not change between inlet and exit. The main program uses three subroutines, RUNGE, RNUMBR, and BOUNCE to trace the particle trajectory in the same manner as discussed previously. The subroutine BOUNCE that is used with this program is the unmodified routine that has been improved considerably. The improved subroutine is the one that has been described previously.

Input

There are two sets of input. The first set specifies the nature of the gas flow and consists of two cards at the front of the data. This program is written so that any consistent system of units may be used. An example of this input data is included with the example case presented after the program listing.

The first two data cards take the following form:

GAM, RGAS, WTFL, RHOIP, TIP	(5F10.6)
D(1), A(1), REXIT, SRDEND .	(4F10.6)

These variables will be defined later.

The second set of data cards specify the type of particle and its initial position and velocity. Each particle can be described by six input data cards which take the following form:

TITLE	(18A4)
VISREF, TREF, TSUT, DGFC	(E20.5, 3F10.3)
RHOP, DIAP, H, TMAX, ETA(1), ETA(2)	(F10.2, 3E10.4, 2F10.3)
YR(I), I = 1,6	(6F10.3)
ALPHA, BETA, GRAVTY	(3F10.4)
NSTEP	(I5)

These variables are defined below. For studies that are done with multiple particles, additional sets of input data cards as indicated for the second data card set may be stacked together. When the program completes the trajectory for one particle, it goes to the next set automatically.

Variables

GAM - Ratio of specific heats

RGAS - Gas constant (Length^2/Time^2 Degree Abs.)

WTFL - Mass flow rate of gas (Mass/Time)

RHOIP - Gas inlet stagnation density (Mass/Length^3)

TIP - Gas inlet stagnation temperature (Degree Abs.)

D(1) - Dimension of the first station as indicated in Figure 20. (Length)

A(1) - Area of the first station. (Length^2)

REXIT - Radial distance that determines exit location. See Figure 20. (Length)

SRDEND - Angular location of the last portion of the regular scroll contour before suppression of the scroll begins. See Figure 20. (Degrees)

TITLE - The first card is reproduced at the top of the first page of output for each particle. Any statement in columns 2 to 72 will be reproduced.

VISREF - Reference viscosity corresponding to TREF. Used in Sutherland's Law. (Mass/Length x Time)

TREF - Reference temperature corresponding to VISREF. Used in Sutherland's Law. (Degrees Abs.)

TSUT - Constant used in Sutherland's Law. (198.6°R or 110°K)

DGFC - Drag Factor - The spherical drag coefficient based on Reynolds Number is multiplied by DGFC. Except in very special cases, this should be 1.0.

RHOP - Particle density. (Mass/Length^3)

DIAP - Particle diameter. (Length)

H - Integration time increment. (Time). If extremely long computer run times are experienced, this can be made larger. If the program fails to converge, this can be made smaller.

TMAX - The program stops the solution if time exceeds TMAX. (Time)

ETA(1) - Normal restitution coefficient.

ETA(2) - Tangential restitution coefficient.

YR(1) - Particle's initial radial position. (Length)

YR(2) - Particle's initial radial velocity. Positive in the outward direction, negative in the inward direction. (Length/Time)

YR(3) - Particle's initial angular position. (Degrees)

YR(4) - Particle's initial angular velocity. (Time)

YR(5) - Particle's initial axial position. (Length)

YR(6) - Particle's initial axial velocity. (Length/Time)

Alpha - Angle of z-axis with respect to the horizonal. See Figure 18. (Degrees)

Beta - Angle of the turbine vertical axis with respect to the gravitational vertical. See Figure 18. (Degrees)

GRAVTY - Acceleration of gravity. (Length/Time^2)

NSTEP - Integer that determines the amount of printed output. Output data is printed every NSTEP time increments.

After initializing the input variables, the program calculates the velocity components and properties of the gas. The gas flow in the scroll is isotropic, with both the mass flow rate and the scroll cross sectional area decreasing uniformly by the same ratio. The velocity is based on the inlet area, but is deflected inward by the angle CH1, where

$$CH1 = \arctan\left(\frac{D(1)}{2\pi\ REXIT}\right)$$

Output

The first part of the output is an echo check of the first set of data cards that describe the gas flow. Such data checks are useful in correcting key punch mistakes on the input cards.

The output variables that are not defined in the input are:

V - Gas velocity (Length/Time)

VR - Radial Component of gas velocity (Length/Time)

VU - Tangential component of gas velocity (Length/Time)

CH1 - Angle between V and VU. (Degrees)

VCR - Critical Velocity. (Length/Time)

RHO - Gas density (Mass/Length^3)

TEMP - Gas temperature (Degrees Abs.)

ORTHOGONAL - Number associated with various radial stations along the scroll.

THETA(I,1) - Angular position of inner contour. (Degrees)

THETA(I,2) - Angular position of outer contour. (Degrees)

R(I,1) - Radial position of inner contour. (Length)

R(I,2) - Radial position of outer contour. (Length).

The program next skips to a new page and the first part of the printed output is an echo check of the input data as punched on the data cards. After initializing the variables, the program calculates and prints several similarity parameters that are useful in relating this particle to other particles that will have similar trajectories. Next the program writes the trajectory information every NSTEP time increments.

The additional terms of the output that are not defined as part of the input are listed below.

DELTA - The characteristic length as given in Equation (7). (Length)

TAU - The time constant as given in Equation (8). (Time)

RECR - The Reynolds number as given in Equation (9).

L - A time increment counter.

T - Time. (Time)

RENOLD - The particle Reynolds number at this point.

Program Listing.

```
G LEVEL  21                    MAIN                    DATE = 75045          15/17/25

C
C    PARTICLE TRAJECTORY SOLUTION IN A SCROLL
C
      DIMENSION STATE(18),YR(6),YRS(6),R(37,2),FR(6),X(2),Y(2),ETA(2)
      DIMENSION A(37),D(37),THETA(37,2)
      DIMENSION AA(3),B(3),C(3),P(3),PP(3),VP(3),PB(3)
      INTEGER RUNGE
      READ(5,1010) GAM,RGAS,WTFL,RHOIP,TIP
      WRITE(6,2010)GAM,RGAS,WTFL,RHOIP,TIP
      READ(5,1020) D(1),A(1),REXIT,SRDEND
      WRITE(6,2020)D(1),A(1),REXIT,SRDEND
C
C   INITIALIZE AND BUILD ARRAYS
C
      DA=A(1)/36.0
      DO 20 I=2,37
      A(I)=A(I-1)-DA
      IF(A(I).GT.1.0E-12) D(I)=D(I-1)*SQRT(A(I)/A(I-1))
      IF(A(I).LE.1.0E-12) D(I)=1.0E-12
      R(I,1)=REXIT
      R(I,2)=REXIT+D(I)
      THETA(I,1)=FLOAT(I-1)*10.
   20 THETA(I,2)=THETA(I,1)
      START=SRDEND/10.+2.
      NSTART=START
      DO 25 I=NSTART,37
      ANG=360.0-THETA(I,2)
   25 R(I,2)=REXIT*SQRT(1.0+TAN(ANG/57.29577)**2)
      R(1,1)=REXIT
      R(1,2)=REXIT+D(1)
      THETA(1,1)=0.0
      THETA(1,2)=0.0
      EXPON=1.0/(GAM-1.0)
      VCR=SQRT(2.0*GAM*RGAS*TIP/(GAM+1.0))
      CIRCUM=2.0*3.1415927*REXIT
      CHI=ATAN2(D(1),CIRCUM)
C
C    DETERMINE GAS VELOCITY
C
      M=1
      VU=WTFL/RHOIP/A(1)
      VR=-VU*TAN(CHI)
      VEST=SQRT(VR**2+VU**2)/VCR
   30 M=M+1
      RHO=RHOIP*(1.0-(GAM-1.0)/(GAM+1.0)*VEST**2)**EXPON
      WTFLES=RHO*A(1)*VU
      IF(ABS(WTFLES-WTFL).LT.1.0E-4) GO TO 40
      IF(M.GT.90) WRITE(6,2060) VEST,RHO,WTFLES,CHI,VU,VR,VCR
      IF(M.GT.101) GO TO 810
      VU=WTFL/RHO/A(1)
      VR=-VU*TAN(CHI)
      VEST=SQRT(VR**2+VU**2)/VCR
      GO TO 30
```

```
   40 V=VEST*VCR
      TEMP=TIP*(1.0-(GAM-1.0)/(GAM+1.0)*VEST**2)
      VZ=0.0
C
C    PRINT OUT SOLUTION.
C
      CHIW=CHI*57.29578
      WRITE(6,2030) V,VR,VU,CHIW,VCR,RHO,TEMP
      DO 50 I=1,37
   50 WRITE(6,2040) I,THETA(I,1),THETA(I,2),R(I,1),R(I,2)
      DO 60 K=1,2
      DO 60 I=1,37
   60 THETA(I,K)=THETA(I,K)/57.29578
      SRDEND=SRDEND/57.29577
      VRSAVE= VR
      VUSAVE=VU
C
C    START PARTICLE SOLUTION
C
  105 READ(5,3000) (STATE(I),I=1,18)
      WRITE(6,4000)(STATE(I),I=1,18)
      READ(5,3010) VISREF,TREF,TSUT,DGFC
      WRITE(6,4010) VISREF,TREF,TSUT,DGFC
      READ(5,3020)  RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      WRITE(6,4020) RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      READ(5,3030) (YR(I),I=1,6)
      WRITE(6,4030)(YR(I),I=1,6)
      YR(3)=YR(3)/57.29577
      READ(5,1000)  ALPHA,BETA,GRAVTY
      WRITE(6,3090) ALPHA,BETA,GRAVTY
      READ(5,3080) NSTEP
      WRITE(6,4080) NSTEP
      RHOCR=RHOIP*(2.0/(GAM  +1.0))**(1.0/(GAM  -1.0))
      DELTA=RHOP*DIAP/RHOCR/0.3
      TCR=(2.0/(GAM  +1.0))*TIP
      VISCR=VISREF*((TCR/TREF)**1.5)*(TREF+TSUT)/(TCR+TSUT)
      TAU=RHOP*DIAP**2/18.0/VISCR
      RECR=DIAP*RHOCR*VCR/VISCR/2.0
      WRITE(6,5020) DELTA,TAU,RECR
      WRITE(6,4090)
C
C    CHECK THAT PARTICLE STARTS INBOUNDS.
C
      NOUT=0
      RW=REXIT+D(1)*SQRT(1.0-YR(3)/6.2831854)
      IF(YR(3).LT.0.0) NOUT=1
      IF(YR(1).GE.RW) NOUT=7
      IF(YR(1).LE.REXIT) NOUT=8
      IF(NOUT.EQ.0) GO TO 130
      WRITE(6,4040) NOUT
      GO TO 105
  130 CONTINUE
C
```

```
C  INITIALIZE FOR FIRST STEP.
C
      ALPHA=ALPHA/57.29577
      BETA=BETA/57.29577
      RPART=DIAP/2.
      N=6
      L=0
      NTIME=NSTEP
      T=0.0
      YR3=YR(3)*57.29577
      WRITE(6,5000) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6)
      TS=T
      DO 620 I=1,N
  620 YRS(I)=YR(I)
      RWS=RW
      VISTAR=VISREF*((TEMP/TREF)**1.5)*(TREF+TSUT)/(TEMP+TSUT)
C
C  INTEGRATE USING RUNGE-KUTTA METHOD.
C
  625 IF(YR(3).LT.SRDEND) GO TO 626
      VR=V*SIN(YR(3))
      VU=V*COS(YR(3))
      GO TO 627
  626 CONTINUE
      VR=VRSAVE
      VU=VUSAVE
  627 CONTINUE
      VDIFF=SQRT((VR-YR(2))**2+(VU-YR(1)*YR(4))**2+(VZ-YR(6))**2)
      RENOLD=RHO*VDIFF*DIAP/VISTAR
      CALL RNUMBR(RENOLD,DGFC,CD)
      BCON=RHO*CD/RHOP/RPART/2.33333
  630 IF(RUNGE(N,YR,FR,T,H).NE.1) GO TO 640
      FR(1)=YR(2)
      FR(2)=YR(1)*YR(4)**2+BCON*VDIFF*(VR-YR(2))-GRAVTY*COS(YR(3)+BETA)
      FR(3)=YR(4)
      FR(4)=-2.0*YR(2)*YR(4)/YR(1)+BCON*VDIFF*(VU-YR(1)*YR(4))/YR(1)
     1  +GRAVTY*SIN(YR(3)+BETA)/YR(1)
      FR(5)=YR(6)
      FR(6)=BCON*VDIFF*(VZ-YR(6))-GRAVTY*SIN(ALPHA)
      GO TO 630
  640 CONTINUE
C
C  DETERMINE IF WALL INTERACTION OCCURRED.
C
      NOUT=0
      IF(YR(3).LT.0.0) NOUT=1
      IF(YR(3).GT.6.2831854) NOUT=3
      IF(YR(1).LE.REXIT) NOUT=8
      IF((NOUT.EQ.1).OR.(NOUT.EQ.3).OR.(NOUT.EQ.8)) GO TO 780
      RW=REXIT+D(1)*SQRT(1.0-YR(3)/6.2831854)
      IF(YR(1).GE.RW) NOUT=7
      IF(YR(3).GT.SRDEND) GO TO 650
      IF(NOUT.EQ.0) GO TO 700
```

```
      NTEST=0
      IF(YR(3).GE.1.57) NTEST=1
      AA(1)=RWS*COS(YRS(3))
      AA(2)=RWS*SIN(YRS(3))
      AA(3)=YRS(5)
      B(1)=RW*COS(YR(3))
      B(2)=RW*SIN(YR(3))
      B(3)=YR(5)
      GO TO 660
  650 XTEST=YR(1)*COS(YR(3))
      IF(XTEST.LT.REXIT) GO TO 700
      NOUT=9
      NTEST=1
      AA(1)=REXIT
      ANG=6.2831854-YRS(3)
      AA(2)=REXIT*TAN(ANG)
      AA(3)=YRS(5)
      B(1)=REXIT
      ANG=6.2831854-YR(3)
      B(2)=REXIT*TAN(ANG)
      B(3)=YR(5)
  660 CONTINUE
      C(1)=AA(1)
      C(2)=AA(2)
      C(3)=YR(5)+ SQRT((AA(1)-B(1))**2+(AA(2)-B(2))**2+(AA(3)-B(3))**2)
      P(1)=YRS(1)*COS(YRS(3))
      P(2)=YRS(1)*SIN(YRS(3))
      P(3)=YRS(5)
      PP(1)=YR(1)*COS(YR(3))
      PP(2)=YR(1)*SIN(YR(3))
      PP(3)=YR(5)
      T=TS
      CALL BOUNCE(AA,B,C,P,PP,H,T,ETA,NFIX,PB,VP)
      TB=ATAN2(PB(2),PB(1))*57.29577
      RB= SQRT(PB(1)**2+PB(2)**2)
      IF((NTEST.EQ.1).AND.(TB.LE.0.0)) TB=TB+360.0
      ZB=PB(3)
      WRITE(6,5030) NOUT,RB,TB,ZB
      YR(1)= SQRT(PB(1)**2+PB(2)**2)
      YR(3)= ATAN2(PB(2),PB(1))
      IF((NTEST.EQ.1).AND.(YR(3).LT.0.0)) YR(3)=YR(3)+6.283184
      YR(5)=PB(3)
      YR(2)=VP(1)*COS(YR(3))+VP(2)*SIN(YR(3))
      YR(4)=-VP(1)*SIN(YR(3))/YR(1)+VP(2)*COS(YR(3))/YR(1)
      YR(6)=VP(3)
  700 L=L+1
      TS=T
      RWS=RW
      DO 760 I=1,N
  760 YRS(I)=YR(I)
      NTIME=NTIME-1
  780 IF((NOUT.EQ.1).OR.(NOUT.EQ.3).OR.(NOUT.EQ.8).OR.(T.GT.TMAX))
     1 GO TO 800
```

```
      IF((ABS(YR(2)).LT.1.0E-4).AND.(ABS(YR(4)).LT.1.0E-4).AND.
     1  (ABS(YR(6)).LT.1.0E-4)) GO TO 800
      IF(NTIME.GT.0) GO TO 625
      NTIME=NSTEP-1
      IF(L.GT.1) NTIME=NSTEP
      YR3=YR(3)*57.29578
      WRITE(6,4050) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6),RENOLD
      GO TO 625
  800 CONTINUE
      YR3=YR(3)*57.29577
      WRITE(6,4060) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6),RENOLD
  805 CONTINUE
      GO TO 105
  810 WRITE(6,2050)
C
C  FORMAT STATEMENTS
C
 1000 FORMAT(7F10.4)
 1010 FORMAT(5F10.6)
 1020 FORMAT(4F10.6)
 2010 FORMAT(1H1,5X,21HINLET SCROLL SOLUTION,//,12X,3HGAM,11X,4HRGAS,
     1  11X,4HWTFL,10X,5HRHOIP,12X,3HTIP,/,(5F15.6))
 2020 FORMAT(1H0,10X,4HD(1),11X,4HA(1),10X,5HPEXIT,9X,6HSRDEND,
     1  /,(4F15.6))
 2030 FORMAT(1H0,13X,1HV,13X,2HVR,13X,2HVU,12X,3HCHI,12X,3HVOR,13X,
     1  3HRHO,11X,4HTEMP,/,(7F15.6),//,11H ORTHOGONAL,5X,10HTHETA(I,1),
     2  5X, 10HTHETA(I,2),9X,6HR(I,1),9X,6HR(I,2))
 2040 FORMAT(I11,2F15.4,2F15.6)
 2050 FORMAT(37HOGAS FLOW SOLUTION FAILED TO CONVERGE)
 2060 FORMAT(1H0,7E15.8)
 3000 FORMAT(18A4)
 3010 FORMAT(E20.5,3F10.3)
 3020 FORMAT(F10.2,3E10.4,2F10.3)
 3030 FORMAT(6F10.3)
 3080 FORMAT(I5)
 3090 FORMAT(1H0,9X,5HALPHA,11X,4HBETA,8X,7HGRAVITY,/,(3F15.4))
 4000 FORMAT(1H1,18A4)
 4010 FORMAT(1H0,13X,6HVISREF,11X,4HTREF,11X,4HTSUT,11X,4HDGEC,/,
     1  (E20.5,3F15.3))
 4020 FORMAT(1H0,10X,4HRHOP,11X,4HDIAP,14X,1HH,11X,4HTMAX,10X,5HETA-N,
     1  10X,5HETA-T,/,(F15.2,3E15.4,2F15.3))
 4030 FORMAT(1H0, 9X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,    5HYR(4),
     1 10X,5HYR(5),10X,5HYR(6),/,(6F15.6))
 4040 FORMAT(1H0,62H PARTICLE NOT IN PASSAGE AT FIRST POINT GIVEN, GO TO
     1 NEXT CASE)
 4050 FORMAT(1H ,I5,E10.2,7F15.5)
 4060 FORMAT(1H0,I5,E10.2,7F15.5)
 4080 FORMAT(17HOPRINT DATA EVERY,I7,2X,7HSTEP(S))
 4090 FORMAT(1H0,3X, 1HL, 9X,1HT,10X,5HYR(1),10X,5HYR(2),10X,5HYR(3),
     1  10X,5HYR(4),10X,5HYR(5),10X,5HYR(6),9X,6HRENOLD)
 5000 FORMAT(1H ,I5,E10.2,6F15.5)
 5020 FORMAT(32HOSIMILARITY PARAMETERS.  DELTA =,F10.4,5X,5HTAU =,F12.4,
     1  5X,6HRECR =,E12.4)

 5030 FORMAT(11H BOUNCE OFF,I5,F15.5,15X,F15.5,15X,F15.5)
      STOP
      END
```

The function routine RUNGE has been removed from the published form of this report to protect the copyright of the authors of Reference 5.

```
      SUBROUTINE RNUMBR(RENOLD,DGFC,CD)
      IF( ABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DGFC*0.4
      RETURN
   26 CD=DGFC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=ALOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1   -0.0065*ARE**5)*DGFC
      RETURN
      END
```

```
      SUBROUTINE BOUNCE(A,B,C,P,PP,H,T,ETA,NFIX,PB,VP)
      DIMENSION A(3),B(3),C(3),P(3),PP(3),V(3),AB(3),AC(3),G(3,3),D(5),
     1  PB(3)
      DIMENSION GS(3,3),VP(3),UN(3) ,PN(3)
      DIMENSION ETA(2)
C
      NFIX=1
      DO 10 I=1,3
      V(I)=(PP(I)-P(I))/H
      AB(I)=B(I)-A(I)
   10 AC(I)=C(I)-A(I)
C
C  DETERMINE UNIT NORMAL TO SURFACE
C
      UN(1)=AB(2)*AC(3)-AB(3)*AC(2)
      UN(2)=AB(3)*AC(1)-AB(1)*AC(3)
      UN(3)=AB(1)*AC(2)-AB(2)*AC(1)
      CMAG= SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
      IF( ABS(CMAG).GT.1.0E-12) GO TO 20
      NFIX=0
      RETURN
   20 DO 30 I=1,3
   30 UN(I)=UN(I)/CMAG
C
C  DETERMINE THE INTERSECTION POINT, PB, OF THE PLANE AND TRAJECTORY.
C
      DETA=UN(1)*V(1)**2+UN(2)*V(1)*V(2)+UN(3)*V(1)*V(3)
      D(1)=UN(1)*A(1)+UN(2)*A(2)+UN(3)*A(3)
      D(2)=V(2)*P(1)-V(1)*P(2)
      D(3)=V(3)*P(1)-V(1)*P(3)
      DO 40 J=1,3
   40 G(1,J)=UN(J)
      G(2,1)= V(2)
      G(2,2)=-V(1)
      G(2,3)=0.0
      G(3,1)= V(3)
      G(3,2)=0.0
      G(3,3)=-V(1)
C
C  IF DETERMINANT EQUALS ZERO, GO TO 80
C
      IF( ABS(DETA).LE.1.0E-12) GO TO 80
      DO 70 K=1,3
      DO 50 I=1,3
      DO 50 J=1,3
   50 GS(I,J)=G(I,J)
      DO 60 I=1,3
   60 GS(I,K)= D(I)
      PB(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
     2 -GS(3,3)*GS(2,1)*GS(1,2)
   70 PB(K)=PB(K)/DETA
      GO TO 100
```

```
C
C  RESET MATRIX AND SOLVE FOR PB.  IF DET. EQUALS ZERO, GO TO 90
C
   80 PB(1)=P(1)
      DETB=-UN(2)*V(2)-UN(3)*V(3)
      IF( ABS(DETB).LE.1.0E-12) GO TO 90
      D(4)=UN(2)*P(2)+UN(3)*P(3)
      D(5)=V(3)*P(2)-V(2)*P(3)
      PB(2)=(-D(4)*V(2)-D(5)*UN(3))/DETB
      PB(3)=(UN(2)*D(5)-V(3)*D(4))/DETB
      GO TO 100
C
C  SPECIAL CASE THAT YIELDS ZERO DETERMINANT ALWAYS.
C
   90 PB(2)=P(2)
      PB(3)=P(3)
  100 CONTINUE
C
C  DETERMINE THE INTERSECTION POINT, PN, OF THE SURFACE NORMAL THRU P
C  IF DETERMINANT EQUALS ZERO, GO TO 140
C
      DETA=UN(1)**3+UN(1)*UN(2)**2+UN(1)*UN(3)**2
      IF( ABS(DETA).LE.1.0E-12) GO TO 140
      D(2)=P(1)*UN(2)-P(2)*UN(1)
      D(3)=P(1)*UN(3)-P(3)*UN(1)
      G(2,1)= UN(2)
      G(2,2)=-UN(1)
      G(2,3)=0.0
      G(3,1)= UN(3)
      G(3,2)=0.0
      G(3,3)=-UN(1)
      DO 130 K=1,3
      DO 110 I=1,3
      DO 110 J=1,3
  110 GS(I,J)=G(I,J)
      DO 120 I=1,3
  120 GS(I,K)=D(I)
      PN(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
     2 -GS(3,3)*GS(2,1)*GS(1,2)
  130 PN(K)=PN(K)/DETA
      GO TO 160
  140 IF(( ABS(UN(1)).GT.1.0E-12).AND.( ABS(UN(2)).GT.1.0E-12))GO TO 150
      PN(1)=P(1)
      PN(2)=P(2)
      PN(3)=P(3)
      PN(3)=A(3)
      GO TO 160
  150 PN(1)=A(1)
      PN(2)=P(2)
      PN(3)=P(3)
  160 CONTINUE
C
```

```
C  DETERMINE PORTION OF TIME SEGMENT USED TO TRAVEL FROM P TO PB.
C
      DPB= SQRT((PB(1)-P(1))**2+(PB(2)-P(2))**2+(PB(3)-P(3))**2)
      VPP=SQRT(V(1)**2+V(2)**2+V(3)**2)
      DTIME=DPB/VPP
C
C  EXTENT LINE PN-PB THE PROPER DISTANCE TO FIND PNP.
C  THEN EXTENT A LINE NORMAL TO THE SURFACE FROM PNP TO GET THE POINT
C  AFTER BOUNCE, PP.
C  FIND VELOCITY COORDINATES BASED ON PP, PB AND TIME REMAINING IN
C  SEGMENT.
C
      DO 170 I=1,3
      S=(PB(I)-PN(I))/DTIME
      PNP=PB(I)+ETA(2)*S*(H-DTIME)
      SN=(PN(I)-P(I))/DTIME
      PP(I)=PNP-ETA(I)*SN*(H-DTIME)
  170 VP(I)=(PP(I)-PB(I))/(H-DTIME)
      T=T+H
      RETURN
      END
```

Example

The example case presented here uses the ft., lbm., second system of units. The gas flow conditions correspond to inlet stagnation conditions of standard sea level air. The scroll dimension D(1) is 0.4 ft. and the exit radius is 0.3615 ft. The particle inlet velocity is in the same direction as the velocity, its magnitude is approximately one half the gas velocity. The particle has a specific gravity of 3 and a diameter of 12 microns. The normal and tangential restitution coefficients are assumed to be 1.0. Gravity is included.

The following pages contain a computer code sheet with the data arranged in the proper columns, and the output for this example.

SCRL 2D Example Case for NASA Report

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

1.4	1715.48	0.486	0.0764	518.7					CARD SET 1
0.4	0.08	0.3615	316.175						
EXAMPLE CASE FØR NASA REPØRT V-PART/V-GAS = 50%									
	0.106E-4	492.0	198.2	1.0					
187.2	0.388E-4	0.1E-4	0.1E5	1.0	1.0				CARD SET 2
0.5615	0.0	0.0001	51.5	0.0	0.0				
0.0	0.0	32.174							
100									
Additional particle trajectories require a Card Set 2 arrangement here.									
These sets are stacked one after the other.									
									CARD SET 2

INLET SCROLL SOLUTION

GAM	RGAS	WTFL	RHOIP	TIP
1.400000	1715.479980	0.486000	0.076400	518.699951

D(1)	A(1)	REXIT	SRDEND
0.400000	0.080000	0.361500	316.174805

V	VR	VU	CHI	VCR	RHO	TEMP
80.950897	-14.039830	79.724136	9.987672	1018.882568	0.076199	518.154053

ORTHOGONAL	THETA(I,1)	THETA(I,2)	R(I,1)	R(I,2)
1	0.0	0.0	0.361500	0.761500
2	10.0000	10.0000	0.361500	0.755905
3	20.0000	20.0000	0.361500	0.750230
4	30.0000	30.0000	0.361500	0.744471
5	40.0000	40.0000	0.361500	0.738623
6	50.0000	50.0000	0.361500	0.732684
7	60.0000	60.0000	0.361500	0.726648
8	70.0000	70.0000	0.361500	0.720510
9	80.0000	80.0000	0.361500	0.714266
10	90.0000	90.0000	0.361500	0.707910
11	100.0000	100.0000	0.361500	0.701434
12	110.0000	110.0000	0.361500	0.694833
13	120.0000	120.0000	0.361500	0.688098
14	130.0000	130.0000	0.361500	0.681221
15	140.0000	140.0000	0.361500	0.674194
16	150.0000	150.0000	0.361500	0.667004
17	160.0000	160.0000	0.361500	0.659641
18	170.0000	170.0000	0.361500	0.652092
19	180.0000	180.0000	0.361500	0.644342
20	190.0000	190.0000	0.361500	0.636373
21	200.0000	200.0000	0.361500	0.628166
22	210.0000	210.0000	0.361500	0.619698
23	220.0000	220.0000	0.361500	0.610943
24	230.0000	230.0000	0.361500	0.601869
25	240.0000	240.0000	0.361500	0.592439
26	250.0000	250.0000	0.361500	0.582607
27	260.0000	260.0000	0.361500	0.572317
28	270.0000	270.0000	0.361500	0.561498
29	280.0000	280.0000	0.361500	0.550060
30	290.0000	290.0000	0.361500	0.537882
31	300.0000	300.0000	0.361500	0.524797
32	310.0000	310.0000	0.361500	0.510569
33	320.0000	320.0000	0.361500	0.471905
34	330.0000	330.0000	0.361500	0.417424
35	340.0000	340.0000	0.361500	0.384700
36	350.0000	350.0000	0.361500	0.367077
37	360.0000	360.0000	0.361500	0.361500

EXAMPLE CASE FOR NASA REPORT V-PART/V-GASS = 50%

VISREF	TREF	TSUT	DGFC
0.10600F-04	492.000	198.200	1.000

RHOP	DIAP	H	TMAX	ETA-N	ETA-T
187.20	0.3880F-04	0.1000E-04	0.1000E-05	1.000	1.000

YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)
0.561500	0.0	0.000100	51.500000	0.0	0.0

ALPHA	BETA	GRAVITY
0.0	0.0	32.1740

PRINT DATA EVERY 100 STEP(S)

SIMILARITY PARAMETERS. DELTA = 0.4999 TAU = 0.1638E-02 RECR = 0.1002E 03

I	T	YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)	RENOLD
0	0.0	0.56150	0.0	0.00010	51.50000	0.0	0.0	
100	0.10F-02	0.55653	-6.90235	5.35006	118.81465	0.0	0.0	4.15392
200	0.20F-02	0.54999	-5.85931	12.79208	137.68571	0.0	0.0	2.44109
300	0.30F-02	0.54486	-4.50850	20.91054	144.63770	0.0	0.0	2.56241
400	0.40F-02	0.54076	-3.76134	29.29121	147.53693	0.0	0.0	2.75175
500	0.50F-02	0.53720	-3.40780	37.79132	149.05623	0.0	0.0	2.84865
600	0.60F-02	0.53388	-3.23498	46.36333	150.13127	0.0	0.0	2.89599
700	0.70F-02	0.53070	-3.13445	54.99220	151.06725	0.0	0.0	2.92322
800	0.80F-02	0.52760	-3.06069	63.67128	151.95792	0.0	0.0	2.94302
900	0.90F-02	0.52457	-2.99645	72.40021	152.83026	0.0	0.0	2.96019
1000	0.10F-01	0.52160	-2.93566	81.17876	153.69142	0.0	0.0	2.97642
1100	0.11F-01	0.51869	-2.87635	90.00636	154.54301	0.0	0.0	2.99224
1200	0.12F-01	0.51584	-2.81796	98.88245	155.38512	0.0	0.0	3.00781
1300	0.13F-01	0.51305	-2.76041	107.80649	156.21758	0.0	0.0	3.02315
1400	0.14F-01	0.51032	-2.70373	116.77803	157.04021	0.0	0.0	3.03825
1500	0.15F-01	0.50764	-2.64802	125.79633	157.85300	0.0	0.0	3.05308
1600	0.16F-01	0.50501	-2.59336	134.86092	158.65587	0.0	0.0	3.06764
1700	0.17F-01	0.50244	-2.53983	143.97130	159.44893	0.0	0.0	3.08188
1800	0.18F-01	0.49993	-2.48751	153.12682	160.23227	0.0	0.0	3.09581
1900	0.19F-01	0.49746	-2.43643	162.32692	161.00612	0.0	0.0	3.10940
2000	0.20F-01	0.49505	-2.38664	171.57117	161.77072	0.0	0.0	3.12265
2100	0.21F-01	0.49269	-2.33815	180.85886	162.52635	0.0	0.0	3.13555
2200	0.22F-01	0.49037	-2.29094	190.18961	163.27335	0.0	0.0	3.14811
2300	0.23F-01	0.48810	-2.24500	199.56297	164.01205	0.0	0.0	3.16034
2400	0.24F-01	0.48587	-2.20028	208.97836	164.74275	0.0	0.0	3.17224
2500	0.25F-01	0.48369	-2.15672	218.43538	165.46588	0.0	0.0	3.18384
2600	0.26F-01	0.48155	-2.11422	227.93378	166.18169	0.0	0.0	3.19516
2700	0.27F-01	0.47946	-2.07271	237.47281	166.89049	0.0	0.0	3.20622
2800	0.28F-01	0.47740	-2.03206	247.05246	167.59248	0.0	0.0	3.21706
2900	0.29F-01	0.47539	-1.99217	256.67188	168.28787	0.0	0.0	3.22770
3000	0.30F-01	0.47341	-1.95290	266.33105	168.97672	0.0	0.0	3.23818
3100	0.31F-01	0.47148	-1.91413	276.02954	169.65903	0.0	0.0	3.24853
3200	0.32F-01	0.46958	-1.87573	285.76709	170.33478	0.0	0.0	3.25878
3300	0.33F-01	0.46772	-1.83758	295.54297	171.00374	0.0	0.0	3.26898
3400	0.34F-01	0.46590	-1.79959	305.35693	171.66573	0.0	0.0	3.27913
3500	0.35F-01	0.46412	-1.76163	315.20874	172.32031	0.0	0.0	3.28929
BOUNCE OFF 9		0.44698		323.97534		0.0		
3600	0.36F-01	0.44470	-56.60704	324.23364	114.17044	0.0	0.0	4.76184
3700	0.37F-01	0.39586	-41.26906	332.46655	169.90005	0.0	0.0	1.60757
3800	0.38F-01	0.36255	-24.94926	343.41602	210.00803	0.0	0.0	0.63308
3804	0.38F-01	0.36132	-24.06871	344.01978	211.60571	0.0	0.0	0.60650

STATOR - Particles in the Stators

This program integrates the equations of motion of a particle in order to determine the trajectory of the particle in a stator. The solution neglects the gravity force, is three dimensional and the particle may bounce off any of the four surfaces that surround the channel. The program is restricted by a constant spacing from hub to shroud. The coordinate system is the r, θ, z system which is indicated in Figure 16.

Method

Figure 21 is a flow diagram for this program. It illustrates the iterative technique used to find the average gas properties along the particle trajectory, which was explained previously. As soon as the program determines that a collision has occurred, the program bounces the particle off the surface and then continues the trajectory from this point.

The main program uses the subroutines, RUNGE, RNUMBR, DLOCAT, POLATE, RESET, BOUNCE and RESTLO. All of these subroutines have been described previously except RESET, which resets the average values used after each iteration, and linearly extrapolates the properties to estimate the average value over the next time step.

Input

There are two sets of input. The first set specifies gas flow properties, and consists of two dimensional arrays for magnitude and direction of the flow velocities within the field. This program works with any consistant system of units. An example of the input data is included with the example case presented after the program listing. The first set of input cards take the following form:

TITLE	(18A4)
MX, KMX, NB	(3I5)

GAMMA, TIP, RHOIP, RGAS	(4F10.6)
VISREF, TREF, TSUT, DGFC	(E20.4, 3F10.6)
ZMAX, FIRST	(2F10.6)
NSTEP, H	(I5, E10.2)
R(I,K) Array	(8F10.6)
THETA(I,K) Array	(8F10.6)
V(I,K) Array	(8F10.6)
BETA(I,K) Array	(8F10.6)

The second set of data cards specify the type of particle, its initial position and velocity. Each particle can be described by three input cards which take the following form.

TITLE	(18A4)
DIAP, RHOIP	(E20.4, F10.6)
(YR(I), I=1,6)	(6F10.6)

These variables are defined below. For studies that are done with multiple particles, additional sets of input data cards as indicated for the second data card set may be stacked together. When the program completes the trajectory for one particle, it goes to the next set automatically.

Variables

TITLE - The first card is reproduced at the top of the first page of output. Any statement in columns 2 to 72 will be reproduced.

MX - The number of blade to blade constant radius orthogonals.

KMX - The number of streamlines used in the description of the flow.

NB - The number of nozzle blades in the ring.

GAMMA - The ratio of specific heats.

TIP - The gas inlet stagnation temperature. (Degrees Abs.)

RHOIP - The gas inlet stagnation density. (Mass/Length^3)

RGAS - The gas constant. (Length^2/Time^2 Degrees Abs.)

VISREF - Reference viscosity corresponding to TREF. Used in Sutherland's Law. (Mass/Length x Time)

TREF - Reference temperature corresponding to VISREF. Used in Sutherland's Law. (Degrees Abs.)

TSUT - Constant used in Sutherland's Law. (198.6°R or 110°K).

DGFC - Drag factor. The spherical drag coefficient based on Reynolds number is multipled by DGFC. Except in very special cases, this should be 1.0.

ZMAX - The flow field extends in the axial direction between z = 0.0 and z = ZMAX. (Length).

FIRST - The angular position of the first blade. (Degrees). See Figure 22. All particles are transposed to a corresponding location in the channel of the first blade, its trajectory determined up till the exit from the blade row. The exit conditions are then moved back to the corresponding location at the original channel.

NSTEP - Integer that determines the amount of printed output. Output data is printed every NSTEP time increments.

H - Integration time increment. (Time). If extremely long computer run times are experienced, this can be made larger. If the program fails to converge, this can be made smaller.

R array - The radial position of the grid points of the flow field. (Length). The program is set up to use constant radius lines from blade to blade, as indicated in Figure 22.

THETA Array - The angular position of the grid points in the flow field. (Degrees). Indicated on Figure 22.

V Array - The normalized gas velocity at the grid point. (V/V_{cr})

BETA Array - The direction of the gas velocity vector at the corresponding grid point. (Degrees). Indicated in Figure 22.

DIAP - Particle diameter. (Length)

RHOP - Particle density. (Mass/Length^3)

YR(1) - Particle initial radial position. (Length)

YR(2) - Particle initial radial velocity. The outward direction corresponds to the positive direction and the inward direction corresponds to the negative direction. (Length/Time)

YR(3) - Particle initial angular position. (Degrees)

YR(4) - Particle initial angular velocity. (1/Time)

YR(5) - Particle initial axial position. (Length)

YR(6) - Particle initial axial velocity. (Length/Time)

OUTPUT

The first part of the output is an echo check of the first set of data cards that describe the gas flow. Such data checks are useful in correcting key punch mistakes on the input cards. These checks cover the first five pages of the output and the four array variables are listed on separate pages. The printed output for each particle starts at the top of a new page with the echo check of the data that corresponds to the data cards for the particle.

Next, the program transposes the initial coordinates of the particle so that the particle enters the first passage. This is done by correcting the angular position of the particle so that the particle is in the required passage. The program notes this correction and writes "PARTICLE ENTERS PASSAGE XX INLET ANGLE CORRECTED TO XXXX". Next, the program calculates the similarity parameters that are useful in relating this particle to other particles that have similar trajectories. Finally, the program writes trajectory information every NSTEP time increments until the solution is complete, and then writes the last solution point and goes on to the next particle.

The additional terms of the output that are not defined as part of the input are listed below.

DELTA - The characteristic length as given in Equation (7). (Length)

TAU - The time constant as given in Equation (8). (Time)

RECR - The Reynolds number as given in Equation (9).

M - An iteration counter. If the air velocity fails to converge to the proper average values after 100 steps, M = 101.

RENOLD - The Reynolds number of the particle at this point.

```
C  PARTICLE TRAJECTORIES IN RADIAL STATOR
C
      INTEGER RUNGE
      DIMENSION R(21,21),THETA(21,21),V(21,21),BETA(21,21),STATE(18),
     1  ETA(2),YR(6),VR(4),VU(4),VZ(4),TEMPA(4),RHOA(4),VISTAR(4),
     2  YRS(6),FR(6),A(3),B(3),C(3),P(3),PP(3),PB(3),VP(3)
C
C  READ FLOW FIELD DATA
C
      READ(5,1030)  (STATE(I),I=1,18)
      WRITE(6,2030) (STATE(I),I=1,18)
      READ(5,1010) MX,KMX,NB
      WRITE(6,2010)MX,KMX,NB
      READ(5,1020)  GAMMA,TIP,RHOIP,RGAS
      WRITE(6,2011) GAMMA,TIP,RHOIP,RGAS
      READ(5,1040) VISREF,TREF,TSUT,DGFC
      WRITE(6,2031)VISREF,TREF,TSUT,DGFC
      READ(5,1020) ZMAX,FIRST
      WRITE(6,2012)ZMAX,FIRST
      READ(5,1050) NSTEP,H
      WRITE(6,2040)NSTEP,H
      WRITE(6,2130)
      DO 10 I=1,MX
      READ(5,1020)  (R(I,K),K=1,KMX)
   10 WRITE(6,2013) (R(I,K),K=1,KMX)
      WRITE(6,2130)
      DO 20 I=1,MX
      READ(5,1020)  (THETA(I,K),K=1,KMX)
   20 WRITE(6,2014) (THETA(I,K),K=1,KMX)
      WRITE(6,2130)
      DO 30 I=1,MX
      READ(5,1020)  (V(I,K),K=1,KMX)
   30 WRITE(6,2015) (V(I,K),K=1,KMX)
      WRITE(6,2130)
      DO 40 I=1,MX
      READ(5,1020) (BETA(I,K),K=1,KMX)
   40 WRITE(6,2016) (BETA(I,K),K=1,KMX)
      VCR=SQRT(2.0*GAMMA*RGAS*TIP/(GAMMA+1.0))
      FIRST=FIRST/57.29577
      DO 60 I=1,MX
      DO 60 K=1,KMX
      THETA(I,K)=THETA(I,K)/57.29577
      BETA(I,K)=BETA(I,K)/57.29577
   60 V(I,K)=V(I,K)*VCR
C
C  READ PARTICLE DATA
C
   50 READ(5,1030)  (STATE(I),I=1,18)
      WRITE(6,2030) (STATE(I),I=1,18)
      READ(5,1040) DIAP,RHOP
      WRITE(6,2032) DIAP,RHOP
      READ(5,1020) (YR(I),I=1,6)
      WRITE(6,2033) (YR(I),I=1,6)
```

```
INITIALIZE

   YR(3)=YR(3)/57.29577
   DO 65 I=1,NB
   TTEST=FIRST+6.283186*FLOAT(I-1)/FLOAT(NB)
   IF(TTEST.GT.YR(3)) GO TO 66
65 CONTINUE
66 J=I-1
   IPASS=J
   YR(3)=YR(3)-FIRST-6.283186*FLOAT(J)/FLOAT(NB)
   YR3=YR(3)*57.29577
   WRITE(6,2140) J,YR3
   EXPON=1.0/(GAMMA-1.0)
   RHOCR=RHOIP*(2.0/(GAMMA+1.0))**EXPON
   DELTA=RHOP*DIAP/RHOCR/0.3
   TCR=TIP*2.0/(GAMMA+1.0)
   VISCR=VISREF*((TCR/TREF)**1.5)*(TREF+TSUT)/(TCR+TSUT)
   TAU=RHOP*DIAP**2/18.0/VISCR
   RECR=DIAP*RHOCR*VCR/VISCR/2.0
   WRITE(6,2050) DELTA,TAU,RECR
   T=0.0
   L=0
   YR3=YR(3)*57.29577
   WRITE(6,2060) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6)
   CP=GAMMA*RGAS/(GAMMA-1.0)
   RPART=DIAP/2.0
   N=6
   M=1
   NTIME=NSTEP
   TS=T
   ALPHA=-90.0/57.29577
   TRPS=TREF+TSUT

DETERMINE AIR VELOCITIES AND PROPERTIES AT PARTICLE LOCATION

   CALL DLOCAT(R,THETA,ZMAX,MX,KMX,YR,IP,KP,NOUT)
   I=(NOUT.EQ.0) GO TO 70
   WRITE(6,2070)
   GO TO 50
70 CALL POLATE(R,THETA,V,YR(1),YR(3),IP,KP,VPP,DD)
   CALL POLATE(R,THETA,BETA,YR(1),YR(3),IP,KP,BETAP,DD)
   VR(1)=VPP*COS(BETAP)*SIN(ALPHA)
   VU(1)=VPP*SIN(BETAP)
   VZ(1)=VPP*COS(BETAP)*COS(ALPHA)
   TEMPA(1)=TIP*(1.0-((GAMMA-1.0)/(GAMMA+1.0))*(VPP/VCR)**2)
   RHOA(1)=RHOIP*(TEMPA(1)/TIP)**EXPON
   VISTAR(1)=VISREF*((TEMPA(1)/TREF)**1.5)*(TRPS)/(TEMPA(1)+TSUT)
   CALL RESET(   VR,1)
   CALL RESET(   VU,1)
   CALL RESET(   VZ,1)
   CALL RESET(TEMPA,1)
   CALL RESET( RHOA,1)
```

```
      CALL RESET(VISTAR,1)
      DO 90 I=1,N
   90 YRS(I)=YR(I)
C
C  INTEGRATE USING RUNGE-KUTTA METHOD
C
  100 VDIFF=SQRT((VR(2)-YR(2))**2+(VU(2)-YR(1)*YR(4))**2
     1   +(VZ(2)-YR(6))**2)
      RENOLD=RHOA(2)*VDIFF*DIAP/VISTAR(2)
      CALL RNUMBR(RENOLD,DGFC,CD)
      BCON=RHOA(2)*CD/RHOP/RPART/2.33333
  110 IF(RUNGE(N,YR,FR,T,H).NE.1) GO TO 120
      FR(1)=YR(2)
      FR(2)=YR(1)*YR(4)**2+BCON*VDIFF*(VR(2)-YR(2))
      FR(3)=YR(4)
      FR(4)=-2.0*YR(2)*YR(4)/YR(1)+BCON*VDIFF*(VU(2)-YR(1)*YR(4))/YR(1)
      FR(5)=YR(6)
      FR(6)=BCON*VDIFF*(VZ(2)-YR(6))
      GO TO 110
  120 CONTINUE
C
C  DETERMINE IF WALL INTERACTION OCCURRED.
C
      CALL DLOCAT(R,THETA,ZMAX,MX,KMX,YR,IP,KP,NOUT)
      IF(NOUT.EQ.0) GO TO 150
      IF((NOUT.EQ.1).OR.(NOUT.EQ.3)) GO TO 200
      GO TO 125
  124 IP=IPS
  125 A(1)=R(IP,KP)*COS(THETA(IP,KP))
      A(2)=R(IP,KP)*SIN(THETA(IP,KP))
      B(1)=R(IP-1,KP)*COS(THETA(IP-1,KP))
      B(2)=R(IP-1,KP)*SIN(THETA(IP-1,KP))
      C(1)=A(1)
      C(2)=A(2)
      IF((NOUT.EQ.5).OR.(NOUT.EQ.6)) GO TO 130
      A(3)=YRS(5)
      B(3)=YRS(5)
      C(3)=YRS(5)+SQRT((A(1)-B(1))**2+(A(2)-B(2))**2+(A(3)-B(3))**2)
      GO TO 140
  130 IF(NOUT.EQ.5) A(3)=0.0
      IF(NOUT.EQ.6) A(3)=ZMAX
      B(3)=A(3)
      C(3)=A(3)
  140 P(1)=YRS(1)*COS(YRS(3))
      P(2)=YRS(1)*SIN(YRS(3))
      P(3)=YRS(5)
      PP(1)=YR(1)*COS(YR(3))
      PP(2)=YR(1)*SIN(YR(3))
      PP(3)=YR(5)
      T=TS
      CALL BOUNCE(A,B,C,P,PP,H,T,ETA,NFIX,PB,VP)
      IF(NFIX.EQ.0) GO TO 200
      IF(NFIX.EQ.2) GO TO 124
```

```
      YR(1)=SQRT(PB(1)**2+PB(2)**2)
      YR(3)=ATAN2(PB(2),PB(1))
      YR3=YR(3)*57.29577
      YR(5)=PB(3)
      WRITE(6,2080) NOUT,YR(1),YR3,YR(5)
      YR(2)=VP(1)*COS(YR(3))+VP(2)*SIN(YR(3))
      YR(4)=(-VP(1)*SIN(YR(3))+VP(2)*COS(YR(3)))/YR(1)
      YR(6)=VP(3)
      GO TO 170
C
C  DETERMINE AIR VALUES AT NEW LOCATION.
C
  150 CALL POLATE(R,THETA,V,YR(1),YR(3),IP,KP,VPP,DD)
      CALL POLATE(R,THETA,BETA,YR(1),YR(3),IP,KP,BETAP,DD)
      VR(4)=VPP*COS(BETAP)*SIN(ALPHA)
      VU(4)=VPP*SIN(BETAP)
      VZ(4)=VPP*COS(BETAP)*COS(ALPHA)
      TEMPA(4)=TIP*(1.0-((GAMMA-1.0)/(GAMMA+1.0))*(VPP/VCR)**2)
      RHOA(4)=RHOIP*(TEMPA(4)/TIP)**EXPON
      VISTAR(4)=VISREF*((TEMPA(4)/TREF)**1.5)*(TRPS)/(TEMPA(4)+TSUT)
C
C  TEST AIR VALUES USED, IF INCORRECT, RESET INTEGRATION VALUES AND
C  GO TO 100.  IF CORRECT, GO TO  170.
C
      IF((ABS(VR(4)-VR(3)).LT.1.0E-4).AND.(ABS(VU(4)-VU(3)).LT.1.0E-4)
     1  .AND.(ABS(VZ(4)-VZ(3)).LT.1.0E-4))GO TO 170
      CALL RESET(    VR,2)
      CALL RESET(    VU,2)
      CALL RESET(    VZ,2)
      CALL RESET( TEMPA,2)
      CALL RESET(  RHOA,2)
      CALL RESET(VISTAR,2)
      T=TS
      DO 160 I=1,6
  160 YR(I)=YRS(I)
      IF(M.GT.100) GO TO 200
      M=M+1
      GO TO 100
C
C  COMPLETE INTEGRATION STEP.
C
  170 M=1
      L=L+1
      IPS=IP
      TS=T
      DO 180 I=1,6
  180 YRS(I)=YR(I)
      CALL RESET(    VZ,3)
      CALL RESET(    VU,3)
      CALL RESET(    VR,3)
      CALL RESET( TEMPA,3)
      CALL RESET(  RHOA,3)
      CALL RESET(VISTAR,3)
```

```
      NTIME=NTIME-1
C
C  IF REQUIRED, WRITE OUTPUT.
C
  190 IF((ABS(YR(2))+ABS(YR(4))+ABS(YR(6))).LT.1.0E-4) GO TO 200
      IF(NTIME.GT.0) GO TO 100
      YR3=YR(3)*57.29577
      WRITE(6,2100) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6),RENOLD
      NTIME=NSTEP-1
      IF(L.GT.1) NTIME=NSTEP
      GO TO 100
  200 CONTINUE
      YR(3)=YR(3)+FIRST+6.2831853/FLOAT(NB)*FLOAT(IPASS)
      YR3=YR(3)*57.29577
      WRITE(6,2110)
      WRITE(6,2100) L,T,YR(1),YR(2),YR3,YR(4),YR(5),YR(6),RENOLD
      WRITE(6,2120) M
      GO TO 50
C
C  FORMAT STATEMENTS.
C
 1010 FORMAT(14I5)
 1020 FORMAT(8F10.6)
 1030 FORMAT(18A4)
 1040 FORMAT(E20.4,3F10.6)
 1050 FORMAT(I5,E10.2)
 2010 FORMAT(1HO,15HFLOW FIELD DATA,//,8X,2HMX,7X,3HKMX,8X,2HNB,/,
     1  (3I10))
 2011 FORMAT(1HO,9X,5HGAMMA,12X,3HTIP,10X,5HRHOIP,11X,4HRGAS,/,(4F15.6))
 2012 FORMAT(1HO,10X,4HZMAX,10X,5HFIRST,/,(2F15.6))
 2013 FORMAT(1H ,4X,6HR(I,K),(8F15.6))
 2014 FORMAT(1H ,10HTHETA(I,K),(8F15.6))
 2015 FORMAT(1H ,4X,6HV(I,K),(8F15.6))
 2016 FORMAT(1H ,1X,9HBETA(I,K),(8F15.6))
 2030 FORMAT(1H1,18A4)
 2031 FORMAT(1HO,13X,6HVISREF,11X,4HTREF,11X,4HTSUT,11X,4HOGFC,/,
     1  (E20.4,3F15.6))
 2032 FORMAT(1HO,15X,4HDIAP,11X,4HRHOP,/,(E20.4,F15.6))
 2033 FORMAT(1HO, 9X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,5HYR(4),10X,
     1  5HYR(5),10X,5HYR(6), /,(6F15.6))
 2040 FORMAT(1HO,4X,5HNSTEP,14X,1HH,/,(I10,E15.2))
 2050 FORMAT(32HOSIMILARITY PARAMETERS.  DELTA =,F10.4,5X,5HTAU =,E12.4,
     1  5X,6HRECR =,E12.4)
 2060 FORMAT(1HO,6X,4HSTEP,9X,1HT,5X,5HYR(1),5X,5HYR(2),5X,5HYR(3),5X,
     1  5HYR(4),5X,5HYR(5),5X,5HYR(6),4X,6HRENOLD, /,
     2  (I11,E10.2,F10.5,F10.2,F10.3,F10.2,F10.5,F10.2))
 2070 FORMAT(1HO,43HPARTICLE OUT OF BOUNDS AT FIRST POINT GIVEN)
 2080 FORMAT(11H BOUNCE OFF,I4,6X,F10.5,10X,F10.3,10X,F10.5)
 2100 FORMAT(1H ,I10,E10.2,F10.5,F10.2,F10.3,F10.2,F10.5,F10.2,E20.4)
 2110 FORMAT(1HO)
 2120 FORMAT(3HOM=,I4)
 2130 FORMAT(1H1)
 2140 FORMAT(24HOPARTICLE ENTERS PASSAGE,I5,5X,24HINLET ANGLE CORRECTED

1TO,(F15.6))
      STOP
      END
```

```
      SUBROUTINE DLOCAT(R,Z,XMAX,MX,KMX,YR,IP,KP,NOUT)
      DIMENSION R(21,21),Z(21,21),YR(6)
      NOUT=0
      DO 20 I=1,MX
      IF(R(I,1).LE.YR(1)) GO TO 30
   20 CONTINUE
      IP=MX
      NOUT=3
      RETURN
   30 IP=I
      IF(IP.NE.1) GO TO 50
      NOUT=1
      RETURN
   50 CONTINUE
      DO 70 K=1,KMX
      X1=R(IP,K)*COS(Z(IP,K))
      Y1=R(IP,K)*SIN(Z(IP,K))
      X2=R(IP-1,K)*COS(Z(IP-1,K))
      Y2=R(IP-1,K)*SIN(Z(IP-1,K))
      PX=YR(1)*COS(YR(3))
      PY=YR(1)*SIN(YR(3))
      IF( ABS(X1-X2).LT.1.0E-12) GO TO 65
      A=(Y1-Y2)/(X1-X2)
      B=Y1-A*X1
      YTEST=A*PX+B
      GO TO 66
   65 YTEST=Y1
   66 CONTINUE
      IF(PY.GE.YTEST) GO TO 80
   70 CONTINUE
      KP=KMX
      NOUT=4
      RETURN
   80 KP=K
      IF(KP.NE.1) GO TO 120
      NOUT=2
      RETURN
  120 IF(YR(5).GT.0.0) GO TO 130
      NOUT=5
      RETURN
  130 IF(YR(5).LT.XMAX) RETURN
      NOUT=6
      RETURN
      END
```

```
      SUBROUTINE RESET(A,I)
      DIMENSION A(4)
C
C        THIS SUBROUTINE RESETS THE VALUES OF A VARIABLE SO THAT THE
C  BEST ESTIMATE OF THE AVERAGE VALUE OF THE VARIABLE CAN BE USED IN
C  THE INTEGRATION STEP.
C  A(1) IS THE VALUE OF A AT POINT 1 FOR THE INTEGRATION STEP.
C  A(2) IS THE AVERAGE VALUE OF A OVER THE INTEGRATION STEP.
C  A(3) IS THE VALUE OF A AT POINT 2 USED IN THIS INTEGRATION STEP.
C  A(4) IS THE VALUE OF A AT POINT 2 CALCULATED AFTER THE INTEGRATION.
C
      GO TO (10,20,30),I
C
C  FOR I=1, SET UP ARRAYS.
C
   10 DO 15 J=2,4
   15 A(J)=A(1)
      RETURN
C
C  FOR I=2, RESET AVERAGE VALUE OF A, A(2), AND REMEMBER THE LATEST
C  VALUE OF A AT THE END OF THE STEP.
C
   20 A(2)=(A(4)+A(1))/2.0
      A(3)=A(4)
      RETURN
C
C  FOR I=3, SYSTEM HAS CONVERGED.  ESTIMATE THE AVERAGE VALUES BY
C  LINEARLY EXTENDING THE VALUES DETERMINED IN THE PREVIOUS POINTS.
C
   30 A(2)=1.5*A(4)-0.5*A(1)
      A(3)=2.0*A(4)-A(1)
      A(1)=A(4)
      A(4)=A(3)
      RETURN
      END
```

The function routine RUNGE has been removed from the published form of this report to protect the copyright of the authors of Reference 5.

```
      SUBROUTINE RESTCO(VN,VT,ETA)
         DIMENSION ETA(2),A(10),B(10)
      DATA A/1.8190,5.1171,-49.2529,131.03528,-174.4249,117.8723,
     1  -24.47885,-16.7297,11.2300,-1.979996/
      DATA B/5.7215,-41.8808,178.1685,-424.3882,572.7631,-406.6625,
     1  87.1428,70.6511,-50.4982,9.6767/
C
C  DATA IN THIS SUBROUTINE CORRESPONDS TO VELOCITIES IN FT/SEC.
C  MATERIAL TYPICAL OF ALUMINUM AND SILICON PARTICLES.
C  ETA(1) IS THE NORMAL RESTITUTION COEFFICIENT.
C  ETA(2) IS THE TANGENTIAL RESTITUTION COEFFICIENT.
C
      BETA=ATAN2(VN,VT)
      V=SQRT(VN**2+VT**2)
      PHIONE=V/250.00
      PHITWO= A(1)*BETA+A(2)*BETA**2+A(3)*BETA**3+A(4)*BETA**4
     1  +A(5)*BETA**5+A(6)*BETA**6+A(7)*BETA**7+A(8)*BETA**8
     2  +A(9)*BETA**9+A(10)*BETA**10
      PHI=PHIONE*PHITWO
      IF(PHI.GT.0.90) PHI=0.90
      ETA(2)=1.000-PHI
      PSIONE=1.0000-EXP(-V/36.000)
      PSITWO= B(1)*BETA+B(2)*BETA**2+B(3)*BETA**3+B(4)*BETA**4
     1  +B(5)*BETA**5+B(6)*BETA**6+B(7)*BETA**7+B(8)*BETA**8
     1  +B(9)*BETA**9+B(10)*BETA**10
      PSI=PSIONE*PSITWO
      IF(PSI.GT.0.900) PSI=0.900
      ETA(1)=1.000-PSI
      RETURN
      END

      SUBROUTINE RNUMBR(RENOLD,DGFC,CD)
      IF( ABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DGFC*0.4
      RETURN
   26 CD=DGFC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=ALOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1  -0.0065*ARE**5)*DGFC
      RETURN
      END
```

```
      SUBROUTINE POLATE(P,Z,A,RP,ZP,IP,KP,AP,DD)
      DIMENSION R(21,21),Z(21,21),A(21,21),D(2),DD(2)
      DO 10 I=1,2
      IA=IP+1-I
      IF( ABS(Z(IA,KP-1)-Z(IA,KP)).LT.1.0E-12) GO TO 5
      IF( ABS(R(IA,KP)-R(IA,KP-1)).LT.1.0E-12) GO TO 6
      AM=(R(IA,KP)-R(IA,KP-1))/(Z(IA,KP)-Z(IA,KP-1))
      B1=R(IA,KP-1)-AM*Z(IA,KP-1)
      B2=RP+ZP/AM
      ZA=(B2-B1)*AM/(AM**2+1.0)
      RA=B2-ZA/AM
      GO TO 10
    5 RA=RP
      ZA=Z(IA,KP-1)
      GO TO 10
    6 RA=R(IA,KP-1)
      ZA=ZP
   10 D(I)= SQRT((RA-RP)**2+(ZA-ZP)**2)
      DT=D(1)+D(2)
      AA=(D(1)*A(IP,KP-1)+D(2)*A(IP-1,KP-1))/DT
      AB=(D(1)*A(IP,KP)+D(2)*A(IP-1,KP))/DT
      DO 20 K=1,2
      KA=KP-K+1
      RC=(D(1)*R(IP-1,KA)+D(2)*R(IP,KA))/DT
      ZC=(D(1)*Z(IP-1,KA)+D(2)*Z(IP,KA))/DT
   20 DD(K)= SQRT((RP-RC)**2+(ZP-ZC)**2)
      DT=DD(1)+DD(2)
      AP=(DD(1)*AA+DD(2)*AB)/DT
      RETURN
      END
```

```
      SUBROUTINE BOUNCE(A,B,C,P,PP,H,T,ETA,NFIX,PB,VP)
      DIMENSION A(3),B(3),C(3),P(3),PP(3),V(3),AB(3),AC(3),G(3,3),D(5),
     1  PB(3)
      DIMENSION GS(3,3),VP(3),UN(3) ,PN(3)
      DIMENSION ETA(2)
      DIMENSION PNP(3),VN(4),VT(4)
C
      NFIX=1
      DO 10 I=1,3
      V(I)=(PP(I)-P(I))/H
      AB(I)=B(I)-A(I)
   10 AC(I)=C(I)-A(I)
      VPP=SQRT(V(1)**2+V(2)**2+V(3)**2)
C  DETERMINE UNIT NORMAL TO SURFACE
C
      UN(1)=AB(2)*AC(3)-AB(3)*AC(2)
      UN(2)=AB(3)*AC(1)-AB(1)*AC(3)
      UN(3)=AB(1)*AC(2)-AB(2)*AC(1)
      CMAG= SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
      IF( ABS(CMAG).GT.1.0E-12) GO TO 20
      NFIX=0
      WRITE(6,1000)
 1000 FORMAT(47H BOUNCE HAS ZERO UNIT VECTOR DESCRIBING SURFACE)
      RETURN
   20 DO 30 I=1,3
   30 UN(I)=UN(I)/CMAG
C
C  DETERMINE THE INTERSECTION POINT, PB, OF THE PLANE AND TRAJECTORY.
C
      DETA=UN(1)*V(1)**2+UN(2)*V(1)*V(2)+UN(3)*V(1)*V(3)
      D(1)=UN(1)*A(1)+UN(2)*A(2)+UN(3)*A(3)
      D(2)=V(2)*P(1)-V(1)*P(2)
      D(3)=V(3)*P(1)-V(1)*P(3)
      DO 40 J=1,3
   40 G(1,J)=UN(J)
      G(2,1)= V(2)
      G(2,2)=-V(1)
      G(2,3)=0.0
      G(3,1)= V(3)
      G(3,2)=0.0
      G(3,3)=-V(1)
C
C  IF DETERMINANT EQUALS ZERO, GO TO 90
C
      IF( ABS(DETA).LE.1.0E-12) GO TO 90
      DO 70 K=1,3
      DO 50 I=1,3
      DO 50 J=1,3
   50 GS(I,J)=G(I,J)
      DO 60 I=1,3
   60 GS(I,K)= D(I)
      PB(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
```

```
     2 -GS(3,3)*GS(2,1)*GS(1,2)
    70 PB(K)=PB(K)/DETA
       GO TO 100
C
C   IF DETERMINANT EQUALS ZERO, POINT P IS ON SURFACE, P EQUALS PB
C
    80 PB(1)=P(1)
       D(4)=D(1)-UN(1)*P(1)
       D(5)=V(3)*P(2)-V(2)*P(3)
       DETA=-UN(2)*V(2)-UN(3)*V(3)
       IF( ABS(DETA).LT.1.0E-12) GO TO 85
       PB(2)=(-D(4)*V(2)-D(5)*UN(3))/DETA
       PB(3)=(UN(2)*D(5)-V(3)*D(4))/DETA
       GO TO 100
    85 IF( ABS(V(3)).GT.1.0E-12) GO TO 90
       PB(2)=P(2)
       PB(3)=A(3)
       GO TO 100
    90 PB(2)=A(2)
       PB(3)=P(3)
   100 CONTINUE
       DPP= SQRT((PP(1)-P(1))**2+(PP(2)-P(2))**2+(PP(3)-P(3))**2)
       DPB= SQRT((PB(1)-P(1))**2+(PB(2)-P(2))**2+(PB(3)-P(3))**2)
       DPPB=SQRT((PP(1)-PB(1))**2+(PP(2)-PB(2))**2+(PP(3)-PB(3))**2)
       IF((DPPB.LT.DPP).AND.(DPB.LT.DPP)) GO TO 103
       NFIX=2
       RETURN
   103 CONTINUE
       IF(DPB.LT.(0.5*DPP)) GO TO 180
C
C   DETERMINE THE INTERSECTION POINT, PN, OF THE SURFACE NORMAL THRU P
C   IF DETERMINANT EQUALS ZERO, GO TO 140
C
       DETA=UN(1)**3+UN(1)*UN(2)**2+UN(1)*UN(3)**2
       IF( ABS(DETA).LE.1.0E-12) GO TO 140
       D(2)=P(1)*UN(2)-P(2)*UN(1)
       D(3)=P(1)*UN(3)-P(3)*UN(1)
       G(2,1)= UN(2)
       G(2,2)=-UN(1)
       G(2,3)=0.0
       G(3,1)= UN(3)
       G(3,2)=0.0
       G(3,3)=-UN(1)
       DO 130 K=1,3
       DO 110 I=1,3
       DO 110 J=1,3
   110 GS(I,J)=G(I,J)
       DO 120 I=1,3
   120 GS(I,K)=D(I)
       PN(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
      1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
      2 -GS(3,3)*GS(2,1)*GS(1,2)
   130 PN(K)=PN(K)/DETA
```

```
      GO TO 160
  140 PN(1)=P(1)
      D(4)=D(1)
      D(5)=UN(3)*P(2)-UN(2)*P(3)
      DETA=-UN(2)**2-UN(3)**2
      PN(2)=(-D(4)*UN(2)-D(5)*UN(3))/DETA
      PN(3)=(UN(2)*D(5)-UN(3)*D(4))/DETA
  160 CONTINUE
C
C  DETERMINE PORTION OF TIME SEGMENT USED TO TRAVEL FROM P TO PB.
C
      DTIME=DPB/VPP
      IF(DTIME.LT.H) GO TO 163
      WRITE(6,1010)
 1010 FORMAT(24H DTIME IS GREATER THAN H)
      NOUT=0
      RETURN
C
C  EXTENT LINE PN-PB THE PROPER DISTANCE TO FIND PNP.
C  THEN EXTENT A LINE NORMAL TO THE SURFACE FROM PNP TO GET THE POINT
C  AFTER BOUNCE, PP.
C  FIND VELOCITY COORDINATES BASED ON PP, PB AND TIME REMAINING IN
C  SEGMENT.
C
  163 DO 165 I=1,3
      VT(I)=(PB(I)-PN(I))/DTIME
  165 VN(I)=(PN(I)-P(I))/DTIME
      VT(4)= SQRT(VT(1)**2+VT(2)**2+VT(3)**2)
      VN(4)= SQRT(VN(1)**2+VN(2)**2+VN(3)**2)
      CALL RESTCO(VN(4),VT(4),ETA)
      DO 170 I=1,3
      PNP(I)=PB(I)+ETA(2)*VT(I)*(H-DTIME)
      PP(I)=PNP(I)-ETA(1)*VN(I)*(H-DTIME)
  170 VP(I)=(PP(I)-PB(I))/(H-DTIME)
      T=T+H
      RETURN
C
C  IF POINT P LIES ON SURFACE, USE POINT PP TO DETERMINE AFTER
C  BOUNCE STATE.
C
  180 CONTINUE
      DETA=UN(1)**3+UN(1)*UN(2)**2+UN(1)*UN(3)**2
      IF( ABS(DETA).LE.1.0E-12) GO TO 220
      D(2)=PP(1)*UN(2)-PP(2)*UN(1)
      D(3)=PP(1)*UN(3)-PP(3)*UN(1)
      G(2,1)= UN(2)
      G(2,2)=-UN(1)
      G(2,3)=0.0
      G(3,1)=UN(3)
      G(3,2)=0.0
      G(3,3)=-UN(1)
      DO 210 K=1,3
      DO 190 I=1,3
```

```
      DO 190 J=1,3
  190 GS(I,J)=G(I,J)
      DO 200 I=1,3
  200 GS(I,K)=D(I)
      PN(K)=GS(1,1)*GS(2,2)*GS(3,3)+GS(1,2)*GS(2,3)*GS(3,1)+GS(1,3)
     1 *GS(2,1)*GS(3,2)-GS(3,1)*GS(2,2)*GS(1,3)-GS(3,2)*GS(2,3)*GS(1,1)
     2 -GS(3,3)*GS(2,1)*GS(1,2)
  210 PN(K)=PN(K)/DETA
      GO TO 240
  220 PN(1)=PP(1)
      D(4)=D(1)
      D(5)=UN(3)*PP(2)-UN(2)*PP(3)
      DETA=-UN(2)*UN(2)-UN(3)*UN(3)
      PN(2)=(-D(4)*UN(2)-D(5)*UN(3))/DETA
      PN(3)=(UN(2)*D(5)-UN(3)*D(4))/DETA
  240 CONTINUE
C
C  DETERMINE PORTION OF TIME SEGMENT REMAINING FOR TRAVEL FROM PB TO PP.
C
      DTIME=H-DPB/VPP
      IF(DTIME.GT.1.0E-12) GO TO 245
      WRITE(6,1020)
 1020 FORMAT(21H DTIME LESS THAN ZERO)
      NFIX=0
      RETURN
C
C  DETERMINE THE PROPER DISTANCE ALONG PN-PB TO FIND PNP.
C  THEN EXTENT A LINE NORMAL TO THE SURFACE FROM PNP TO GET THE POINT
C  AFTER BOUNCE, PP.
C  FIND VELOCITY COORDINATES BASED ON PP, PB, AND THE TIME H.
C
  245 DO 250 I=1,3
      VT(I)=(PN(I)-PB(I))/DTIME
  250 VN(I)=(PP(I)-PN(I))/DTIME
      VT(4)= SQRT(VT(1)**2+VT(2)**2+VT(3)**2)
      VN(4)= SQRT(VN(1)**2+VN(2)**2+VN(3)**2)
      CALL RESTCO(VN(4),VT(4),ETA)
      DO 260 I=1,3
      PNP(I)=PB(I)+ETA(2)*VT(I)*DTIME
      PP(I)=PNP(I)-ETA(1)*VN(I)*DTIME
  260 VP(I)=(PP(I)-PB(I))/DTIME
      T=T+H
      RETURN
      END
```

Example

The example case presented here used the ft., lbm., second system of units. The gas flow conditions correspond to inlet stagnation conditions of standard sea level air. In the output, the R, THETA, V, and BETA arrays have been combined onto one page. The nozzle blades lie between radii of 0.274 ft., and 0.321 ft. The velocities are expressed in terms of V/V_{cr}, as can be seen from the output array.

The particle used in the example has a specific gravity of 3 and a diameter of approximately 24 microns. Initially, the particle has a velocity that is equal to one half the gas velocity. The output for the particle indicates that the particle enters passage 26, but the angular position has been corrected to correspond to the proper position in the passage for which the data on velocity and velocity direction apply.

The trajectory data illustrates a bounce off the pressure surface of the blade, in this case the iteration scheme failed to converge in 100 steps. This can be corrected by making the time step smaller.

The following pages contain a computer code sheet with the data arranged in the proper columns, and the output for this example.

STATOR Example Case for NASA Report

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

RADIAL STATOR CASE FOR THESIS AND NASA REPORT

10 5 29

1.4 518.7 0.0748 1715.48

0.106E-4 492.0 198.2 1.0

0.0264 12.41

10 0.1E-5

0.321 0.321 0.321 0.321 0.321

. . . . .

0.274 0.274 0.274 0.274 0.274

-0.41 -3.60 -6.50 -9.00 -12.81

. . . . .

15.00 12.00 9.50 7.00 4.75

0.142 0.139 0.124 0.107 0.074

. . . . .

0.786 0.786 0.786 0.786 0.786

-12.00 20.00 31.50 39.00 32.00

. . . . .

79.00 77.50 74.80 73.20 72.50

CARD SET 1

EXAMPLE CASE FOR NASA REPORT V-PART/V-GAS = 50%.

CARD SET 2

0.774E-4 187.2

0.3209 -107.84 328.97 205.95 0.0132 0.0

Additional particle trajectories require a Card Set 2 arrangement here.

These sets are stacked one after the other.

CARD SET 2

BARTAN STATOR CASE FOR THESIS AND NASA REPORT

FIRM FIELD DATA

MX	KPX	AR
10	5	29

GAMMA	TIP	RMCIP	RGAS
1.400000	518.699951	0.074800	1715.479980

VISREF	TREF	TSLT	DGFC
0.1060E-04	492.000000	198.199997	1.000000

ZMAX	FIRST
0.026400	12.410000

NSTEP	F
10	0.10E-05

R(I,K)	0.321000	0.321000	0.321000	0.321000	0.321000
R(I,K)	0.319000	0.319000	0.319000	0.319000	0.319000
R(I,K)	0.316000	0.316000	0.316000	0.316000	0.316000
R(I,K)	0.312000	0.312000	0.312000	0.312000	0.312000
R(I,K)	0.308000	0.308000	0.308000	0.308000	0.308000
R(I,K)	0.304000	0.304000	0.304000	0.304000	0.304000
R(I,K)	0.297500	0.297500	0.297500	0.297500	0.297500
R(I,K)	0.291000	0.291000	0.291000	0.291000	0.291000
R(I,K)	0.282500	0.282500	0.282500	0.282500	0.282500
R(I,K)	0.274000	0.274000	0.274000	0.274000	0.274000

THETA(I,K)	-0.410000	-3.599999	-6.500000	-9.000000	-12.813999
THETA(I,K)	-0.750000	-3.200000	-6.000000	-8.799999	-11.580000
THETA(I,K)	-1.160000	-2.099999	-5.799999	-8.099999	-11.000000
THETA(I,K)	-1.160000	-2.900000	-5.099999	-7.500000	-10.330000
THETA(I,K)	-0.750000	-2.200000	-4.299999	-6.700000	-9.419999
THETA(I,K)	-0.330000	-1.599999	-3.599999	-5.900000	-8.419999
THETA(I,K)	0.920000	0.0	-2.000000	-4.000000	-7.500000
THETA(I,K)	3.099999	2.000000	0.0	-2.000000	-4.160000
THETA(I,K)	7.900000	6.000000	4.000000	1.900000	-0.160000
THETA(I,K)	15.000000	12.000000	9.500000	7.000000	4.750000

V(I,K)	0.142000	0.139000	0.124000	0.107000	0.074000
V(I,K)	0.152000	0.146000	0.140000	0.113000	0.098000
V(I,K)	0.214000	0.173000	0.154000	0.126000	0.103000
V(I,K)	0.285000	0.211000	0.182000	0.146000	0.107000
V(I,K)	0.353000	0.263000	0.207000	0.174000	0.123000
V(I,K)	0.418000	0.335000	0.255000	0.215000	0.163000
V(I,K)	0.559000	0.448000	0.373000	0.276000	0.231000
V(I,K)	0.604000	0.582000	0.495000	0.404000	0.306000
V(I,K)	0.676000	0.634000	0.634000	0.581000	0.535000
V(I,K)	0.786000	0.786000	0.786000	0.786000	0.786000

PETA(I,K)	-17.000000	20.000000	31.500000	39.000000	32.000000
PETA(I,K)	-41.799988	20.199997	34.500000	40.000000	44.199997
PETA(I,K)	-20.000000	57.500000	31.500000	42.000000	43.500000
PETA(I,K)	14.500000	30.699997	42.000000	45.599991	48.000000
PETA(I,K)	29.500000	38.000000	45.000000	48.500000	52.000000
PETA(I,K)	36.399994	45.099991	48.000000	51.199997	55.000000
PETA(I,K)	53.000000	54.000000	55.000000	56.000000	59.799988
PETA(I,K)	67.000000	61.000000	61.000000	62.000000	64.199997
PETA(I,K)	72.500000	71.000000	69.000000	68.000000	68.500000
PETA(I,K)	79.000000	77.500000	74.799988	73.199997	72.500000

EXAMPLE CASE FOR NASA REPORT V-FAST/V-GAS = SC1.

FIAP	RHCP
0.7740E-04	187.199997

YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)
0.320900	-107.835556	338.569531	205.949997	0.013200	0.0

PARTICLE ENTERS PASSAGE 26 INLET ANGLE CORRECTED TO -6.158586

SIMILARITY PARAMETERS. DELTA = 1.0185 T/L = 0.6520E-02 RECP = 0.1957E 03

STEP	T	YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)	RENOLD
0	0.0	0.32C9C	-107.84	-6.199	205.95	0.01320	0.0	
10	0.10E-04	0.31982	-107.73	-6.080	207.42	0.01320	0.00	0.3972E 01
20	0.20E-04	0.31874	-107.61	-5.961	208.93	0.01320	0.00	0.1700E 02
30	0.30E-04	0.31767	-107.53	-5.841	210.66	0.01320	0.00	0.1356E 02
40	0.40E-04	0.31659	-107.44	-5.719	212.32	0.01320	0.00	0.1065E 02
50	0.50E-04	0.31552	-107.40	-5.597	214.28	0.01320	0.00	0.3201E 02
60	0.60E-04	0.31445	-107.40	-5.474	216.57	0.01320	0.00	0.2725E 02
70	0.70E-04	0.31337	-107.37	-5.349	218.69	0.01320	0.00	0.2235E 02
80	0.80E-04	0.31230	-107.30	-5.223	220.65	0.01320	0.00	0.1757E 02
90	0.90E-04	0.31122	-107.26	-5.096	223.45	0.01320	0.00	0.4311E 02
100	0.10E-03	0.31015	-107.42	-4.967	226.39	0.01320	0.00	0.3843E 02
110	0.11E-03	0.30908	-107.44	-4.837	229.13	0.01320	0.00	0.3378E 02
120	0.12E-03	0.30800	-107.42	-4.705	231.68	0.01320	0.00	0.2916E 02
130	0.13E-03	0.30692	-107.67	-4.571	235.98	0.01320	0.00	0.6097E 02
140	0.14E-03	0.30585	-107.65	-4.434	239.98	0.01320	0.00	0.5336E 02
150	0.15E-03	0.30477	-107.56	-4.296	243.59	0.01320	0.00	0.4587E 02
160	0.16E-03	0.30369	-108.18	-4.155	248.17	0.01320	0.00	0.1164E 03
170	0.17E-03	0.30260	-108.80	-4.011	256.17	0.01320	0.00	0.1039E 03
180	0.18E-03	0.30151	-109.30	-3.862	263.29	0.01320	0.00	0.9191E 02
190	0.19E-03	0.30042	-109.68	-3.709	269.62	0.01320	0.00	0.8051E 02
200	0.20E-03	0.29932	-109.94	-3.553	275.23	0.01320	0.00	0.6977E 02
210	0.21E-03	0.29822	-110.09	-3.394	280.20	0.01320	0.00	0.5972E 02
220	0.22E-03	0.29712	-110.42	-3.232	287.62	0.01320	0.00	0.1556E 03
230	0.23E-03	0.29601	-111.14	-3.068	299.64	0.01320	0.00	0.1356E 03
240	0.24E-03	0.29489	-111.71	-2.898	310.35	0.01320	0.00	0.1236E 03
250	0.25E-03	0.29377	-112.12	-2.708	319.79	0.01320	0.00	0.1079E 03
260	0.26E-03	0.29265	-112.38	-2.522	328.01	0.01320	0.00	0.9176E 02
270	0.27E-03	0.29153	-112.49	-2.332	335.14	0.01320	0.00	0.7762E 02
280	0.28E-03	0.29040	-112.98	-2.131	348.96	0.01320	0.00	0.2103E 03
290	0.29E-03	0.28926	-113.73	-1.932	367.65	0.01320	0.00	0.1935E 03
300	0.30E-03	0.28812	-114.22	-1.716	384.69	0.01320	0.00	0.1756E 03
310	0.31E-03	0.28698	-114.73	-1.491	400.05	0.01320	0.00	0.1566E 03
320	0.32E-03	0.28583	-114.94	-1.258	413.68	0.01320	0.00	0.1365E 03
330	0.33E-03	0.28468	-114.95	-1.018	425.53	0.01320	0.00	0.1153E 03
REINCE CEE	4	0.28378		-0.828		0.01320		
340	0.34E-03	0.28376	-13.31	-0.778	417.21	0.01320	-0.02	0.1200E 03
350	0.35E-03	0.28362	-14.13	-0.537	424.36	0.01320	-0.02	0.1204E 03
360	0.36E-03	0.28347	-14.96	-0.292	431.59	0.01320	-0.02	0.1214E 03
370	0.37E-03	0.28332	-15.79	-0.043	438.91	0.01320	-0.02	0.1224E 03
380	0.38E-03	0.28316	-16.63	0.211	446.32	0.01320	-0.02	0.1232E 03
390	0.39E-03	0.28299	-17.49	0.469	453.81	0.01320	-0.02	0.1243E 03
400	0.40E-03	0.28281	-18.35	0.731	461.38	0.01320	-0.02	0.1252E 03
410	0.41E-03	0.28262	-19.22	0.998	469.04	0.01320	-0.02	0.1262E 03
420	0.42E-03	0.28242	-20.49	1.265	483.81	0.01320	-0.02	0.2896E 03
430	0.43E-03	0.28221	-22.39	1.554	511.38	0.01320	-0.02	0.2836E 03
435	0.43E-03	0.28209	-23.29	336.671	524.95	0.01320	-0.02	0.2866E 03

P= 101

ROTOR - Particles in the Rotor

This program calculates the trajectory of a particle in a radial inflow turbine rotor. The gas flow solution is based on the quasi-orthogonal method, of Reference 9. The program that was given in Reference 9 is modified to provide required output on data cards. To avoid confusion, a listing of the modified program is included here as Part A of the Program Listing.

Method

The flow diagram of the program is given in Figure 23. The results of the fluid solutions are stored in arrays that specify the gas velocity vectors at the intersection points of the quasi-orthogonals and streamlines. With the fluid solution arrays input, the program goes to a RUNGE-KUTTA technique to integrate the three-dimensional equations of motion of the particle.

The integration of the equations of motion over one time increment, is first carried out using the velocity and the gas properties at point A to determine the new particle location B. The program then calls ALOCAT and POLATE to determine the gas velocity components at B_1. A corrected gas velocity components are calculated from B_1 with the mean values at A and B_1. The program then integrates the equations of motion again starting from A to find the corrected particle location B_2. ALOCAT and POLATE are called again to give the gas velocity components at B_2, and these velocity components are compared to the corresponding values at B_1, if the difference is large, the previous iteration is repeated. Once the iteration has converged, the trajectory to point B has been determined and this point is used as the initial point for the next time increment.

The subroutine ALOCAT is used to determine the subscripts of the grid points that surround the particle. Figure 15 shows a typical particle within a set of quasi-orthogonals and streamlines. The subroutine returns the values IP and KP that locate the particle within a particular grid. The subroutine also returns JP, which is the next higher subscript in the XT and THTA arrays. If the particle

is no longer within the boundaries of the flow field, the subroutine returns NOUT which is a code specifying where the particle has gone out of the flow field.

The subroutine POLATE interpolates the value of any variable whose values are known at four grid points surrounding the particle. Referring to Figure 15, the subroutine first calculates the distances D(1) and D(2), and based on these distances, determines a weighted average of the variable at two locations on adjacent streamlines. These values are AA and AB. Then the subroutine determines DD(1) and DD(2) and uses these distances to get the weighted average of AA and AB at the position occupied by the particle.

The subroutine RBCH considers the particles that rebound from the casing or the hub. It is called whenever the particle position B is outside the casing or the hub boundaries. The subroutine returns to the previous position, and linearly extends the trajectory over one time increment, with the bounce occurring at some portion of the time segment. The subroutines writes "BOUNCE OFF SURFACE (NOUT) ..." and prints the location of the bounce.

The subroutine RBBB considers the case where the particle rebounds from the blade surfaces. The procedure is the same as RBCH.

The subroutine RNUMBR is used to determine the drag coefficients based on a curve fit of the drag versus Reynolds Number data. Equations 20 are used, and Figure 3 demonstrates the fit of these equations to data.

The subroutine RUNGE uses a fourth order method to integrate a system of simultaneous first order ordinary differential equations across one time step. Reference 5 explains this subroutine in more detail.

Input

The input cards take the following format. Units consistent with the quasi-orthogonal program as given in Reference 9 must be used.

The first group of input cards are the punched output cards from the quasi-orthogonal program of Reference 9. These cards are punched in the correct format when the code BCDP in the quasi-orthogonal program is set equal to 2. An example of the input data is included with the example case presented after the program listing.

Following these, the data sets corresponding to the particle trajectories are

TITLE	(18A4)
VISREF, TREF, TSUT, DGFC	(E20.5, 3F10.3)
RHOP, DIAP, H, TMAX, ETA-N, ETA-T	(F10.2, 3E10.4, 2F10.3)
YR(I), I = 1, 6	(6F10.3)
NSTEP	(I5)

Multiple sets of this group may be stacked together for cases of more than one particle. The input variables are defined below.

TITLE - The first card is reproduced at the top of the first page of output for each particle. Any statement in columns 2 through 72 will be reproduced.

VISREF - Reference viscosity corresponding to TREF. Used in Sutherland's Law. (lbm/ft sec).

TREF - Reference temperature corresponding to VISREF. Used in Sutherland's Law. (^{o}R).

TSUT - Constant used in Sutherland's Law. (198.6^{o}R).

DGFC - Drag factor. The drag coefficient based on Reynold's Number is multiplied by DGFC. Except in very special cases, this should be 1.0.

RHOP - Particle density. (lbm/ft^{3}).

DIAP - Particle diameter. (Ft).

H - Time increment used in the integration process. (Sec).

TMAX - Program stops if time exceeds TMAX. (Sec).

ETA-N - Normal restitution coefficients.

ETA-T - Tangential restitution coefficients.

YR(1) - Initial radial position of the particle. (Ft).

YR(2) - Initial radial velocity component of the particle. (Ft/Sec).

YR(3) - Initial angular position of the particle. (Radians).
YR(4) - Initial angular velocity of the particle. (Sec^{-1}).
YR(5) - Initial axial velocity of the particle. (Ft).
YR(6) - Initial axial velocity of the particle. (Ft/Sec).
NSTEP - The program prints out trajectory information every NSTEP time increments.

Output

An example listing of typical output is included after the program listing. This program output can be divided into several groups.

Output Group A.

This set of output is a reprint of some of the data that is transferred from the quasi-orthogonal solution.

Output Group B.

This set of output is concerned with the particle trajectory. The first part is an echo check of the data cards corresponding to a particle. Such data checks are useful in correcting key punch mistakes on the input cards.

After initializing the variables, the program calculates and prints several similarity parameters that are useful in relating this particle to other particles having similar trajectories. The quantities that are printed are explained below.

STEP - A count of each of the integration steps.
T - Time. (Sec).
YR(1) ... YR(6) - Position and velocity components of the particle with respect to the rotor.
ABS - The angular position of the particle with respect to the absolute reference frame.
REYNOLD - Reynolds Number of the particle at this location.

When the particle leaves either the exit or inlet of the flow field, the program prints the last data information and includes the last value of STEP and M. The last M is the count of the number of iterations required for the solution of particle location to converge. If M is greater than 100, the program truncates prematurely.

```
C
C     USE OF ARBITRARY QUASI-ORTHOGONALS FOR CALCULATING FLOW
C    DISTRIBUTION IN THE MERIDIONAL PLANE OF A TURBOMACHINE.
C     THEODORE KATSANIS  NASA TECHNICAL NOTE D-2546     DEC. 1964
C
      COMMON SRW
      DIMENSION AL(21,21),BETA(21,21),CAL(21,21),CBETA(21,21),
     1CURV(21,21),DN(21,21),PRS(21,21),R(21,21),Z(21,21),SM(21,21),
     2SA(21,21),SB(21,21),SC(21,21),SD(21,21),SAL(21,21),SBETA(21,21),
     3TN(21,21),TT(21,21),WA(21,21),WTR(21,21)
      DIMENSION AB(21),AC(21),AD(21),BA(21),DELBTA(21),DRDM(21),
     1DTDR(21),DTDZ(21),DWMDM(21),DWTDM(21),RH(21),RS(21),ZH(21),ZS(21),
     2THTA(21),WTFL(21),XR(21),XT(21),XZ(21)
      INTEGER RUNO,TYPE,BCDP,SRW
      RUNO = 0
   10 READ(5,1010) MX,KMX,MR,MZ,W,WT,XN,GAM,AR
      ITNO = 1
      RUNO = RUNO + 1
      WRITE(6,1020) RUNO
      WRITE(6,1010) MX,KMX,MR,MZ,W,WT,XN,GAM,AR
      READ(5,1010)  TYPE,BCDP,SRW,NULL,TEMP,ALM,RHO,TOLER,PLOSS
      WRITE(6,1010) TYPE,BCDP,SRW,NULL,TEMP,ALM,RHO,TOLER,PLOSS
      READ(5,1010)  MTHTA,NPRT,ITER,NULL,SFACT,ZSPLIT,BETIN,RB,CORFAC
      WRITE(6,1010) MTHTA,NPRT,ITER,NULL,SFACT,ZSPLIT,BETIN,RB,CORFAC
      READ(5,1011)  WTOLER
      WRITE(6,1011) WTOLER
      READ(5,1030)  (ZS(I),I=1,MX)
      WRITE(6,1030) (ZS(I),I=1,MX)
      READ(5,1030)  (ZH(I),I=1,MX)
      WRITE(6,1030) (ZH(I),I=1,MX)
      READ(5,1030)  (RS(I),I=1,MX)
      WRITE(6,1030) (RS(I),I=1,MX)
      READ(5,1030)  (RH(I),I=1,MX)
      WRITE(6,1030) (RH(I),I=1,MX)
      DO 20 I = 1,MX
      ZS(I)=ZS(I)/12.
      ZH(I)=ZH(I)/12.
      RS(I)=RS(I)/12.
   20 RH(I)=RH(I)/12.
      IF(TYPE.NE.0) GO TO 40
      WA(1,1)=WT/RHO/(ZS(1)-ZH(1))/3.14/(RS(1)+RH(1))
      DO 30 I = 1,MX
      DN(I,KMX)=SQRT((ZS(I)-ZH(I))**2+(RS(I)-RH(I))**2)
      DO 30 K = 1,KMX
      DN(I,K) = FLOAT(K-1)/FLOAT(KMX-1)*DN(I,KMX)
      WA(I,K) = WA(1,1)
      Z(I,K)=DN(I,K)/DN(I,KMX)*(ZS(I)-ZH(I))+ZH(I)
   30 R(I,K)=DN(I,K)/DN(I,KMX)*(RS(I)-RH(I))+RH(I)
      GO TO 50
   40 IF(TYPE.NE.1) GO TO 145
      DO 45 I=1,MX
      READ(5,7010)  (DN(I,K),K=1,KMX)
      READ(5,7010)  (WA(I,K),K=1,KMX)
```

```
      READ(5,7010)  ( Z(I,K),K=1,KMX)
      READ(5,7010)  ( R(I,K),K=1,KMX)
   45 CONTINUE
      WRITE(6,1040)
   50 READ  (5,1030)(THTA(I),I=1,MTHTA)
      WRITE (6,1030)(THTA(I),I=1,MTHTA)
      READ  (5,1030)(XT(I),I=1,MTHTA)
      WRITE (6,1030)(XT(I),I=1,MTHTA)
      DO 60 K=1,MR
      READ  (5,1030)(TN(I,K),I=1,MZ)
   60 WRITE (6,1030)(TN(I,K),I=1,MZ)
      READ  (5,1030)(XZ(I),I=1,MZ)
      WRITE (6,1030)(XZ(I),I=1,MZ)
      READ  (5,1030)(XR(I),I=1,MR)
      WRITE (6,1030)(XR(I),I=1,MR)
C
C END OF INPUT STATEMENTS.
C
C
C SCALING - CHANGE INCHES TO FEET AND PSI TO LB/SQ.FT,
C INITIALIZE, CALCULATE CONSTANTS.
C
   70 DO 90 K = 1,MR
      DO 80 I = 1,MZ
   80 TN(I,K) = TN(I,K)/12.0
   90 XR(K) = XR(K)/12.0
      DO 100 I = 1,MZ
  100 XZ(I) = XZ(I)/12.0
      DO 110 K = 1,KMX
  110 SM(1,K) = 0.0
      BA(1) = 0.0
      DO 120 K = 2,KMX
  120 BA(K)=FLOAT(K-1)*WT/FLOAT(KMX-1)
      DO 130 I = 1,MX
  130 DN(I,1) = 0.0
      DO 140 I = 1,MTHTA
  140 XT(I) = XT(I)/12.0
      ROOT=SQRT(2.0)
  145 CONTINUE
      DO 146 I=1,MX
      DO 146 K=1,KMX
  146 WTR(I,K)=0.0
      TOLER = TOLER/12.0
      RB = RB/12.0
      ZSPLIT = ZSPLIT/12.0
      PLOSS = PLOSS*144.0
      CI = SQRT(GAM*AR*TEMP)
      WRITE (6,1050) CI
      KMXM1 = KMX-1
      CP=AR*GAM/(GAM-1.)
      EXPON = 1.0/(GAM-1.0)
      BETIN = -BETIN/57.29577
      RINLET = (RS(1)+RH(1))/2.0
```

```
      CEF = SIN(BETIN)/COS(BETIN)/RINLET/(RINLET-RB)**2
      ERROR = 100000.0
C
C BEGINNING OF LOOP FOR ITERATIONS.
C
  150 IF(ITER.EQ.0) WRITE(6,1060) ITNO
      IF(ITER.EQ.0) WRITE(6,1070)
      ERROR1 = ERROR
      ERROR = 0.0
C
C START OF CALCULATIONS OF PARAMETERS.
C
      DO 230 K = 1,KMX
      DO 160 I = 1,MX
      AB(I) = (Z(I,K)-R(I,K))/ROOT
  160 AC(I) = (Z(I,K)+R(I,K))/ROOT
      CALL SPLINE(AB,AC,MX,AL(1,K),CURV(1,K))
      DO 170 I = 1,MX
      CURV(I,K) = CURV(I,K)/(1.+AL(I,K)**2)**1.5
      AL(I,K) = ATAN(AL(I,K))-0.785398
      CAL(I,K) = COS(AL(I,K))
  170 SAL(I,K) = SIN(AL(I,K))
      DO 180 I = 2,MX
  180 SM(I,K)=SM(I-1,K)+SQRT((Z(I,K)-Z(I-1,K))**2+(R(I,K)-R(I-1,K))**2)
  190 CALL SPLDER(XT(1),THTA(1),MTHTA,Z(1,K),MX,DTDZ(1))
      DO 220 I = 1,MX
      CALL LININT(Z(I,K),R(I,K),XZ,XR,TN,21,21,T)
      IF(R(I,K).LE.RB) GO TO 200
      DTDR(I)=CEF*(R(I,K)-RB)**2
      GO TO 210
  200 DTDR(I) = 0.0
  210 TQ = R(I,K)*DTDR(I)
      TP = R(I,K)*DTDZ(I)
      TT(I,K) = T*SQRT(1.+TP*TP)
      BETA(I,K) = ATAN(TP*CAL(I,K)+TQ*SAL(I,K))
      SBETA(I,K)=SIN(BETA(I,K))
      CBETA(I,K)=COS(BETA(I,K))
      SA(I,K)=CBETA(I,K)**2*CAL(I,K)*CURV(I,K)-SBETA(I,K)**2/R(I,K)
     1+SAL(I,K)*CBETA(I,K)*SBETA(I,K)*DTDR(I)
      SC(I,K)=-SAL(I,K)*CBETA(I,K)**2*CURV(I,K)+SAL(I,K)*CBETA(I,K)
     1*SBETA(I,K)*DTDZ(I)
      AB(I)= WA(I,K)*CBETA(I,K)
  220 AC(I)=WA(I,K)*SBETA(I,K)
      CALL SPLINE(SM(1,K),AB,MX,DWMDM,AD)
      CALL SPLINE(SM(1,K),AC,MX,DWTDM,AD)
      IF((ITER.LE.0).AND.(MOD(K-1,NPRT).EQ.0)) WRITE(6,1080) K
      DO 230 I = 1,MX
      SB(I,K)=SAL(I,K)*CBETA(I,K)*DWMDM(I)-2.*W*SBETA(I,K)+DTDR(I)
     1*R(I,K)*CBETA(I,K)*(DWTDM(I)+2.*W*SAL(I,K))
      SD(I,K)=CAL(I,K)*CBETA(I,K)*DWMDM(I)+DTDZ(I)*R(I,K)*CBETA(I,K)
     1*(DWTDM(I)+2.*W*SAL(I,K))
      IF((ITER.GT.0).OR.(MOD(K-1,NPRT).NE.0)) GO TO 230
      A = AL(I,K)*57.29577
```

```
      B = SM(I,K)*12.
      E = TT(I,K)*12.
      G = BETA(I,K)*57.29577
      WRITE(6,1090) A,CURV(I,K),B,G,E,SA(I,K),SB(I,K),SC(I,K),SD(I,K)
  230 CONTINUE
C
C END OF LOOP - PARAMETER CALCULATION.
C
C
C CALCULATE BLADE SURFACE VELOCITIES. (AFTER CONVERGENCE.)
C
      IF(ITER.NE.0) GO TO 260
      DO 250 K = 1,KMX
      CALL SPLINE(SM(1,K),TT(1,K),MX,DELBTA,AC)
      A=XN
      DO 240 I = 1,MX
  240 AB(I)=(R(I,K)*W+WA(I,K)*SBETA(I,K))*(6.283186*R(I,K)/A-TT(I,K))
      CALL SPLINE(SM(1,K),AB,MX,DRDM,AC)
      IF(SFACT.LE.1.0) GO TO 245
      A = SFACT*XN
      DO 244 I = 1,MX
  244 AB(I)=(R(I,K)*W+WA(I,K)*SBETA(I,K))*(6.283186*R(I,K)/A-TT(I,K))
      CALL SPLINE(SM(1,K),AB,MX,AD,AC)
  245 DO 250 I = 1,MX
      BETAD=BETA(I,K)-DELBTA(I)/2.
      BETAT = BETAD+DELBTA(I)
      COSBD = COS(BETAD)
      COSBT = COS(BETAT)
      IF(Z(I,K).LT.ZSPLIT) DRDM(I) = AD(I)
      WTR(I,K)=COSBD*COSBT/(COSBD+COSBT)*(2.*WA(I,K)/COSBD+R(I,K)*W
     1*(BETAD-BETAT)/CBETA(I,K)**2+DRDM(I))
  250 CONTINUE
C
C END OF BLADE SURFACE VELOCITY CALCULATIONS.
C
C
C START CALCULATION OF WEIGHT FLOW VS. DISTANCE FROM HUB.
C
  260 DO 370 I = 1,MX
      IND = 1
      DO 270 K = 1,KMX
  270 AC(K)=DN(I,K)
      GO TO 290
  280 WA(I,1)=0.5*WA(I,1)
  290 DO 300 K = 2,KMX
      J = K-1
      HR = R(I,K)-R(I,J)
      HZ = Z(I,K)-Z(I,J)
      WAS=WA(I,J)*(1.+SA(I,J)*HR+SC(I,J)*HZ)+SB(I,J)*HR+SD(I,J)*HZ
      WASS=WA(I,J)+WAS*(SA(I,K)*HR+SC(I,K)*HZ)+SB(I,K)*HR+SD(I,K)*HZ
  300 WA(I,K)=(WAS+WASS)/2.
  310 DO 340 K = 1,KMX
      TIP=1.-(WA(I,K)**2+2.*W*ALM-(W*R(I,K))**2)/2./CP/TEMP
```

```
      IF(T1P.LT.0.0) GO TO 280
      TPP1P=1.-(2.*W*ALM-(W*R(I,K))**2)/2./CP/TEMP
      DENSTY=T1P**EXPON*RHO-(T1P/TPP1P)**EXPON*PLOSS/AR/TPP1P/TEMP
     1*32.17*SM(I,K)/SM(MX,K)
      PRS(I,K)=DENSTY*AR*T1P*TEMP/32.17/144.
      IF(ZS(I).LE.ZH(I)) GO TO 320
      PSI=ATAN((RS(I)-RH(I))/(ZS(I)-ZH(I)))-1.5708
      GO TO 330
  320 PSI=ATAN((ZH(I)-ZS(I))/(RS(I)-RH(I)))
  330 WTHRU = WA(I,K)*CBETA(I,K)*COS(PSI-AL(I,K))
      A = XN
      IF(Z(I,K).LT.ZSPLIT) A = SFACT*XN
      C=6.283186*R(I,K)-A*TT(I,K)
  340 AD(K)=DENSTY*WTHRU*C
      CALL INTGRL(AC(1),AD(1),KMX,WTFL(1))
      IF(ABS(WT-WTFL(KMX)).LE.WTOLER) GO TO 350
      CALL CONTIN(WA(I,1),WTFL(KMX),IND,I,WT)
      IF(IND.NE.6) GO TO 290
  350 CALL SPLINT(WTFL,AC,KMX,BA,KMX,AB)
      DO 360 K = 1,KMX
      DELTA=ABS(AB(K)-DN(I,K))
      DN(I,K)=(1.-CORFAC)*DN(I,K)+CORFAC*AB(K)
  360 IF(DELTA.GT.ERROR) ERROR = DELTA
  370 CONTINUE
C
C END OF LOOP - WEIGHT FLOW CALCULATION.
C
C
C CALCULATE STREAMLINE COORDINATES FOR NEXT ITERATION.
C
      DO 380 K = 2,KMXM1
      DO 380 I = 1,MX
      Z(I,K)=DN(I,K)/DN(I,KMX)*(ZS(I)-ZH(I))+ZH(I)
  380 R(I,K)=DN(I,K)/DN(I,KMX)*(RS(I)-RH(I))+RH(I)
      IF((ERROR.GE.ERROR1).OR.(ERROR.LE.TOLER)) ITER=ITER-1
      IF(ITER.GT.0) GO TO 410
      WRITE(6,1100)
      DO 400 K = 1,KMX,NPRT
      WRITE (6,1080) K
      DO 390 I = 1,MX
      AB(I) = (Z(I,K)-R(I,K))/ROOT
  390 AC(I) = (Z(I,K)+R(I,K))/ROOT
      CALL SPLINE(AB,AC,MX,AL(1,K),CURV(1,K))
      DO 400 I = 1,MX
      CURV(I,K)=CURV(I,K)/(1.+AL(I,K)**2)**1.5
      A=DN(I,K)*12.
      B=Z(I,K)*12.
      D=R(I,K)*12.
  400 WRITE (6,1110)A,B,D,WA(I,K),PRS(I,K),WTR(I,K),CURV(I,K)
      WRITE (6,1130)
  410 A =ERROR*12.
      WRITE (6,1120) ITNO,A
      ITNO = ITNO+1
```

```
      IF(ITER.GE.0) GO TO 150
      IF(BCDP.NE.1) GO TO 415
      DO 414 I=1,MX
      PUNCH 7010, (DN(I,K),K=1,KMX)
      PUNCH 7010, (WA(I,K),K=1,KMX)
      PUNCH 7010, ( Z(I,K),K=1,KMX)
      PUNCH 7010, ( R(I,K),K=1,KMX)
  414 CONTINUE
  415 IF(BCDP.NE.2) GO TO 10
C
C
C  CARD SET FOR READING AND WRITING BETWEEN QUASI-ORTHOGONAL PROGRAM
C  AND TRAJECTORIES PROGRAM.  WRITE SET.
C
      PUNCH 7000, MX,KMX,MTHTA
      PUNCH 7010, GAM,TEMP,RHO,AR,SFACT
      PUNCH 7010, ZSPLIT,W,XN,ALM,PLOSS
      DO 7777 I=1,MX
      PUNCH 7010, (   R(I,K),K=1,KMX)
      PUNCH 7010, (   Z(I,K),K=1,KMX)
      PUNCH 7010, (  WA(I,K),K=1,KMX)
      DO 777 K=1,KMX
  777 AL(I,K)=ATAN(AL(I,K))-0.7853982
      PUNCH 7010, (  AL(I,K),K=1,KMX)
      PUNCH 7010, (BETA(I,K),K=1,KMX)
      PUNCH 7010, (  SM(I,K),K=1,KMX)
 7777 CONTINUE
      PUNCH 7010, (THTA(I),I=1,MTHTA)
      PUNCH 7010, (  XT(I),I=1,MTHTA)
 7000 FORMAT(7I10)
 7010 FORMAT(5E14.6)
C
  420 GO TO 10
C
C  END OF CALCULATION OF NEXT STREAMLINE COORDINATES.
C
C  FORMAT STATEMENTS.
C
 1010 FORMAT(4I5,5F10.4)
 1011 FORMAT(F10.5)
 1020 FORMAT(1H1,7HRUN NO.,I3,10X,23HINPUT DATA CARD LISTING)
 1030 FORMAT(7F10.4)
 1040 FORMAT(1H ,10X,25H BCD CARDS FOR DN,WA,Z,R.)
 1050 FORMAT(1H1,4X,31HSTAG. SPEED OF SOUND AT INLET =,F9.2)
 1060 FORMAT(1H0,//,5X,13HITERATION NO.,I3)
 1070 FORMAT(1H ,6X,2HAL,12X,2HRC,12X,2HSM,12X,4HBETA,10X,2HTT,12X,2HSA,
     1 12X,2HSB,12X,2HSC,12X,2HSD)
 1080 FORMAT(1H ,2X,10HSTREAMLINE,I3)
 1090 FORMAT(9F14.6)
 1100 FORMAT(1H1,9X,2HDN,18X,1HZ,19X,1HR,19X,2HWA,18X,3HPRS,16X,3HWTR,
     1 14X,2HRC)
 1110 FORMAT(1H ,6F19.6,F18.6)
 1120 FORMAT(1H ,4X,13HITERATION NO.,I3,10X,24HMAX. STREAMLINE CHANGE =,
     1 F10.6)
 1130 FORMAT(1H1)
C
C  END OF FORMAT STATEMENTS.
      END
```

```
      INTEGER RUNGE
      DIMENSION R(21,21),Z(21,21),WA(21,21),AL(21,21),BETA(21,21),
     1  TT(21,21),THTA(21),XT(21)
      DIMENSION STATE(18),YR(6),YRS(6),WR(4),WU(4),WZ(4),RHOA(4),
     1  TEMPA(4),VISTAR(4),ETA(2),DD(2),AR(2),AZ(2)
      DIMENSION SM(21,21),FR(6),RA(2),ZA(2),THETA(2)
C
C  READ QUASI-ORTHOGONAL RESULTS - WRITE SIGNIFICANT PARTS.
C
      CALL BAREAD(MX,KMX,MTHTA)
      WRITE(6,4040) MX,KMX,MTHTA
      CALL BDREAD(GAMMA,TEMP,RHO)
      CALL BDREAD(RGAS,SFACT,ZSPLIT)
      CALL BDREAD(W,XN,ALM)
      CALL BDREAD(PLOSS,ANULL,BNULL)
      WRITE(6,4050) GAMMA,TEMP,RHO,RGAS,SFACT,ZSPLIT,W
      WRITE(6,4090) XN,ALM,PLOSS
      CALL BCREAD(R)
      CALL BCREAD(Z)
      CALL BCREAD(WA)
      CALL BCREAD(AL)
      CALL BCREAD(BETA)
      CALL BCREAD(SM)
      CALL BBREAD(THTA)
      CALL BBREAD(XT)
      WRITE(6,4070)
      DO 10 I=1,MX
   10 WRITE(6,4060) R(I,1),Z(I,1),R(I,KMX),Z(I,KMX)
      WRITE(6,5050) (XT(I),I=1,MTHTA)
      WRITE(6,5040) (THTA(I),I=1,MTHTA)
C
C  READ PARTICLE DATA
C
  605 READ(5,3000) (STATE(I),I=1,18)
      WRITE(6,4000) (STATE(I),I=1,18)
      READ(5,3010) VISREF,TREF,TSUT,DGFC
      WRITE(6,4010)VISREF,TREF,TSUT,DGFC
      READ(5,3020)  RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      WRITE(6,4020) RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      READ(5,3030)  (YR(I),I=1,6)
      WRITE(6,4030) (YR(I),I=1,6)
      READ(5,5010) NSTEP
      WRITE(6,5020) NSTEP
C
C  INITIALIZE.
C
      T=0.0
      L=0
      WRITE(6,5000) L,T,(YR(I),I=1,6)
      EXPON=1.0/(GAMMA-1.0)
      CP=GAMMA*RGAS/(GAMMA-1.0)
      RPART=DIAP/2.0
      N=6
```

```
      M=1
      NTIME=NSTEP
      TS=T
C
C  DETERMINE AIR VELOCITIES AND PROPERTIES AT PARTICLE LOCATION.
C  SET UP TO START TRACE, INITIALIZE.
C
      CALL ALCCAT(R,Z,MX,KMX,YR,THTA,XT,MTHTA,XN,ZSPLIT,IP,JP,KP,NOUT)
      IF(NOUT.EQ.0) GO TO 606
      WRITE(6,4080)
      GO TO 605
C
  606 CALL POLATE(R,Z,WA,YR(1),YR(5),IP,KP,WAP,DD)
      CALL POLATE(R,Z,BETA,YR(1),YR(5),IP,KP,BETAP,DD)
      CALL POLATE(R,Z,AL,YR(1),YR(5),IP,KP,ALPP,DD)
      CALL POLATE(R,Z,SM,YR(1),YR(5),IP,KP,SMP,DD)
      WR(1)=WAP*COS(BETAP)*SIN(ALPP)
      WU(1)=WAP*SIN(BETAP)
      WZ(1)=WAP*COS(BETAP)*COS(ALPP)
      SMT=(DD(1)*SM(MX,KP)+DD(2)*SM(MX,KP-1))/(DD(1)+DD(2))
      TIP=1.0-(WAP**2+2.*W*ALM-(W*YR(1))**2)/2.0/CP/TEMP
      TPPIP=1.0-(2.*W*ALM-(W*YR(1))**2)/2.0/CP/TEMP
      RHOA(1)=TIP**EXPON*RHO-(TIP/TPPIP)**EXPON*PLOSS/RGAS/TPPIP/TEMP
     1 *32.17*SMP/SMT
      TEMPA(1)=TIP*TEMP
      VISTAR(1)=VISREF*((TEMPA(1)/TREF)**1.5)*(TREF+TSUT)/(TEMPA(1)
     1  +TSUT)
C
C  INITIALIZE FOR FIRST STEP.
C
      DO 610 I=1,4
      WR(I)=WR(1)
      WU(I)=WU(1)
      WZ(I)=WZ(1)
      RHOA(I)=RHOA(1)
      TEMPA(I)=TEMPA(1)
  610 VISTAR(I)=VISTAR(1)
      DO 620 I=1,N
  620 YRS(I)=YR(I)
C
C  INTEGRATE USING RUNGE-KUTTA METHOD
C
  625 VDIFF=SQRT((WR(2)-YR(2))**2+(WU(2)-YR(1)*YR(4))**2
     1  +(WZ(2)-YR(6))**2)
      RENOLD=RHOA(2)*VDIFF*DIAP/VISTAR(2)
      CALL RNUMBR(RENOLD,DGFC,CD)
      BCON=RHOA(2)*CD/PHOP/RPART/2.33333
  630 IF(RUNGE(N,YR,FR,T,H).NE.1) GO TO 640
      FR(1)=YR(2)
      FR(2)=YR(1)*YR(4)**2+2.0*YR(1)*W*YR(4)+YR(1)*W**2
     1  +BCON*VDIFF*(WR(2)-YR(2))
      FR(3)=YR(4)
      FR(4)=-2.0*YR(2)*YR(4)/YR(1)-2.0*W*YR(2)/YR(1)
```

C-2

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     1  +BCON*VDIFF*(WU(2)-YR(1)*YR(4))/YP(1)
        ER(5)=YR(6)
        ER(6)=BCON*VDIFF*(WZ(2)-YR(6))
        GO TO 630
  640 CONTINUE
C
C  DETERMINE IF WALL INTERACTION OCCURRED.
C
        CALL ALCCAT(R,Z,MX,KMX,YR,THTA,XT,MTHTA,XN,ZSPLIT,IP,JP,KP,NOUT)
        IF(NOUT.EQ.0) GO TO 700
        IF(NOUT.EQ.1) GO TO 780
        IF(NOUT.EQ.3) GO TO 780
        DO 645 I=1,6
  645 YR(I)=YRS(I)
        T=TS
        IF(NOUT.NE.2) GO TO 650
        RA(1)=R(IP,1)
        RA(2)=R(IP-1,1)
        ZA(1)=Z(IP,1)
        ZA(2)=Z(IP-1,1)
        CALL RBCH(YR,RA,ZA,T,H,ETA,NCUT)
        GO TO 750
  650 IF(NOUT.NE.4) GO TO 660
        RA(1)=R(IP,KMX)
        RA(2)=R(IP-1,KMX)
        ZA(1)=Z(IP,KMX)
        ZA(2)=Z(IP-1,KMX)
        CALL RBCH(YR,RA,ZA,T,H,ETA,NOUT)
        GO TO 750
  660 THETA(1)=THTA(JP)
        THETA(2)=THTA(JP-1)
        ZA(1)=XT(JP)
        ZA(2)=XT(JP-1)
        IF(YR(5).GE.ZSPLIT) DTHET=3.1415927/XN
        IF(YR(5).LT.ZSPLIT) DTHET=1.5707963/XN
        CALL RBBB(YR,THETA,ZA,T,H,ETA,NOUT,DTHET)
        GO TO 750
C
C  DETERMINE AIR VALUES AT NEW LOCATION
C
  700 CALL POLATE(R,Z,WA     ,YR(1),YR(5),IP,KP,WAP     ,DD)
        CALL POLATE(R,Z,BETA   ,YR(1),YR(5),IP,KP,BETAP   ,DD)
        CALL POLATE(R,Z,AL     ,YR(1),YR(5),IP,KP,ALPP    ,DD)
        CALL POLATE(R,Z,SM     ,YR(1),YR(5),IP,KP,SMP     ,DD)
        WR(4)=WAP*COS(BETAP)*SIN(ALPP)
        WU(4)=WAP*SIN(BETAP)
        WZ(4)=WAP*COS(BETAP)*COS(ALPP)
        SMT=(DD(1)*SM(MX,KP)+DD(2)*SM(MX,KP-1))/(DD(1)+DD(2))
        TIP=1.0-(WAP**2+2.*W*ALM-(W*YR(1))**2)/2.0/CP/TEMP
        TPPIP=1.0-(2.*W*ALM-(W*YR(1))**2)/2.0/CP/TEMP
        RHOA(4)=TIP**EXPON*RHO-(TIP/TPPIP)**EXPON*PRESS/RGAS/TPPIP/TEMP
     1  *32.17*SMP/SMT
        TEMPA(4)=TIP*TEMP
```

```
      VISTAR(4)=VISREF*((TEMPA(4)/TREF)**1.5)*(TREF+TSUT)/(TEMPA(4)
     1  +TSUT)
C
C  TEST AIR VALUES USED, IF INCORRECT,RESET INTEGRATION VALUES AND GO
C  625.    IF CORRECT, GO TO 750.
C
      IF((ABS(WR(4)-WR(3)).LT.1.0E-4).AND.(ABS(WU(4)-WU(3)).LT.1.0E-4)
     1  .AND.(ABS(WZ(4)-WZ(3)).LT.1.0E-4)) GO TO 750
      WR(2)=(WR(4)+WR(1))/2.0
      WU(2)=(WU(4)+WU(2))/2.0
      WZ(2)=(WZ(4)+WZ(1))/2.0
      WR(3)=WR(4)
      WU(3)=WU(4)
      WZ(3)=WZ(4)
      RHOA(2)=(RHOA(4)+RHOA(1))/2.0
      TEMPA(2)=(TEMPA(4)+TEMPA(1))/2.0
      VISTAR(2)=(VISTAR(4)+VISTAR(2))/2.0
      VISTAR(2)=(VISTAR(4)+VISTAR(1))/2.0
      T=TS
      DO 710 I=1,N
  710 YP(I)=YPS(I)
      IF(M.GT.100) GO TO 800
      M=M+1
      GO TO 625
C
C  COMPLETE INTEGRATION STEP.  IF REQUIRED, WRITE OUTPUT.  GO TO 625
C
  750 M=1
      L=L+1
      TS=T
      DO 760 I=1,N
  760 YPS(I)=YP(I)
      WR(2)=1.5*WR(4)-0.5*WR(1)
      WR(3)=2.0*WR(4)-WR(1)
      WR(1)=WR(4)
      WR(4)=WR(3)
      WU(2)=1.5*WU(4)-0.5*WU(1)
      WU(3)=2.0*WU(4)-WU(1)
      WU(1)=WU(4)
      WU(4)=WU(3)
      WZ(2)=1.5*WZ(4)-0.5*WZ(1)
      WZ(3)=2.0*WZ(4)-WZ(1)
      WZ(1)=WZ(4)
      WZ(4)=WZ(3)
      RHOA(2)=1.5*RHOA(4)-0.5*RHOA(1)
      RHOA(3)=2.0*RHOA(4)-RHOA(1)
      RHOA(1)=RHOA(4)
      RHOA(4)=RHOA(3)
      TEMPA(2)=1.5*TEMPA(4)-0.5*TEMPA(1)
      TEMPA(3)=2.0*TEMPA(4)-TEMPA(1)
      TEMPA(1)=TEMPA(4)
      TEMPA(4)=TEMPA(3)
      VISTAR(2)=1.5*VISTAR(4)-0.5*VISTAR(1)
```

```
      VISTAR(3)=2.0*VISTAR(4)-VISTAR(1)
      VISTAR(1)=VISTAR(4)
      VISTAR(4)=VISTAR(3)
      NTIME=NTIME-1
  780 IF((NOUT.EQ.1).OR.(NOUT.EQ.3).OR.(T.GT.TMAX)) GO TO 800
      IF(NTIME.GT.0) GO TO 625
      WRITE(6,2050) L,T,(YR(I),I=1,6),RENOLD
      NTIME=NSTEP-1
      IF(L.GT.1) NTIME=NSTEP
      GO TO 625
  800 CONTINUE
      WRITE(6,2050) L,T,(YR(I),I=1,6),RENOLD
      WRITE(6,2060) M,L
  805 CONTINUE
      GO TO 605
 2050 FORMAT(1H ,I10,F15.6,7F15.5)
 2060 FORMAT(1H0,2I10)
 3000 FORMAT(18A4)
 3010 FORMAT(F20.4,3F10.4)
 3020 FORMAT(F10.4,F10.3,F10.3,3F10.4)
 3030 FORMAT(6F10.5)
 4000 FORMAT(1H1,18A4)
 4010 FORMAT(1H0,13X,6HVISREF,11X,4HTREF,11X,4HTSUT,11X,4HRGFC,/,
     1  (F20.4,3F15.4))
 4020 FORMAT(1H0,10X,4HRHOP,11X,4HDIAP,14X,1HH,11X,4HTMAX,10X,5HETA-N,
     1  10X,5HETA-T,/,(F15.4,2F15.3,3F15.4))
 4030  FORMAT(1H0, 9X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,5HYR(4),10X,
     1  5HYR(5),10X,5HYR(6),/,(6F15.5))
 4040 FORMAT(1H1,44HIMPORTANT DATA FROM QUASI-ORTHOGONAL PROGRAM,//,
     1  8X,2HMX,7X,3HKMX,5X,5HMTHTA,/,(3I10))
 4050 FORMAT(1H0, 9X,5HGAMMA,11X,4HTEMP,12X,3HRHO,11X,4HRGAS,10X,
     1  5HSFACT, 9X,6HZSPLIT,14X,1HW,/,(7F15.4))
 4060 FORMAT(1H ,2F15.6,5X,2F15.6)
 4070 FORMAT(1H0,15HHUB COORDINATES,27X,15HTIP COORDINATES,/,7X,1HR,
     1 14X,1HZ,19X,1HR,14X,1HZ)
 4080 FORMAT(46HPARTICLE OUT OF PASSAGE AT FIRST POINT GIVEN.)
 4090 FORMAT(1H0,12X,2HXN,12X,3HALM,10X,5HPLOSS,/,
     1  (3F15.4))
 5000 FORMAT(1H0,5X,4HSTEP,14X,1HT,10X,5HYR(1),10X,5HYR(2),10X,5HYR(3),
     1  10X,5HYR(4),10X,5HYR(5),10X,5HYR(6),9X,6HRENOLD,
     1  /,(I11,E15.6,6F15.5))
 5010 FORMAT(I5)
 5020 FORMAT(17HOPRINT DATA EVERY,I7,2X,7HSTEP(S))
 5040 FORMAT(12HOTHETA ARRAY,/,(6F15.5))
 5050 FORMAT(9HOXT ARRAY,/,(6F15.5))
      END
```

```
      SUBROUTINE ALOCAT(R,Z,MX,KMX,YR,THTA,XT,MTA,XN,ZST,IP,JP,KP,NOUT
      DIMENSION R(21,21),Z(21,21),RC(21),ZC(21),YR(6),THTA(21),XT(21)
      MTHTA=MTA
      ZSPLIT=ZST
      NOUT=0
      DO 20 I=1,MX
      IF(ABS(Z(I,1)-Z(I,KMX)).LT.1.0E-12) GO TO 10
      A=(R(I,1)-R(I,KMX))/(Z(I,1)-Z(I,KMX))
      B=R(I,1)-A*Z(I,1)
      RTEST=A*YR(5)+B
      IF(RTEST.LE.YR(1)) GO TO 30
      GO TO 20
   10 IF(Z(I,1).GE.YR(5)) GO TO 30
   20 CONTINUE
   30 IP=I
      IF(IP.NE.1) GO TO 40
      NOUT=1
      RETURN
   40 IF(IP.NE.MX) GO TO 50
      IF(Z(MX,1).GT.YR(5)) GO TO 50
      NOUT=3
      RETURN
   50 CONTINUE
      DO 70 K=1,KMX
      IF(ABS(Z(IP,K)-Z(IP-1,K)).LT.1.0E-12) GO TO 60
      A=(R(IP-1,K)-R(IP,K))/(Z(IP-1,K)-Z(IP,K))
      B=R(IP,K)-A*Z(IP,K)
      RTEST=A*YR(5)+B
      IF(YR(1).LE.RTEST) GO TO 80
      GO TO 70
   60 IF(Z(IP,K).GE.YR(5)) GO TO 80
   70 CONTINUE
   80 KP=K
      IF(KP.NE.1) GO TO 90
      NOUT=2
      RETURN
   90 IF(KP.NE.KMX) GO TO 120
      IF(ABS(Z(IP,KP)-Z(IP-1,KP)).LT.1.0E-12) GO TO 110
      IF(YR(1).LT.RTEST) GO TO 120
      NOUT=4
      RETURN
  110 IF(YR(5).LE.Z(IP,KP)) NOUT=4
      RETURN
  120 CONTINUE
      DO 130 J=1,MTHTA
      IF(XT(J).GE.YR(5)) GO TO 140
  130 CONTINUE
  140 JP=J
      A=(THTA(JP-1)-THTA(JP))/(XT(JP-1)-XT(JP))
      B=THTA(JP)-A*XT(JP)
      THTB=YR(5)*A+B
      IF(YR(5).GE.ZSPLIT) DTHTA=3.1415927/XN
      IF(YR(5).LT.ZSPLIT) DTHTA=1.5707963/XN
```

```
      THET1=DTHTA+THTB
      THET2=-DTHTA+THTB
      IF(YR(3).LT.THET1) GO TO 150
      NOUT=5
      RETURN
  150 IF(YR(3).GT.THET2) GO TO 160
      NOUT=6
  160 RETURN
      END


      SUBROUTINE POLATE(R,Z,A,RP,ZP,IP,KP,AP,DD)
      DIMENSION R(21,21),Z(21,21),A(21,21),D(2),DD(2)
      DO 10 I=1,2
      IA=IP+1-I
      IF( ABS(Z(IA,KP-1)-Z(IA,KP)).LT.1.0E-12) GO TO 5
      IF( ABS(R(IA,KP)-R(IA,KP-1)).LT.1.0E-12) GO TO 6
      AM=(R(IA,KP)-R(IA,KP-1))/(Z(IA,KP)-Z(IA,KP-1))
      B1=R(IA,KP-1)-AM*Z(IA,KP-1)
      B2=RP+ZP/AM
      ZA=(B2-B1)*AM/(AM**2+1.0)
      RA=B2-ZA/AM
      GO TO 10
    5 RA=RP
      ZA=Z(IA,KP-1)
      GO TO 10
    6 RA=R(IA,KP-1)
      ZA=ZP
   10 D(I)= SQRT((RA-RP)**2+(ZA-ZP)**2)
      DT=D(1)+D(2)
      AA=(D(1)*A(IP,KP-1)+D(2)*A(IP-1,KP-1))/DT
      AB=(D(1)*A(IP,KP)+D(2)*A(IP-1,KP))/DT
      DO 20 K=1,2
      KA=KP-K+1
      RC=(D(1)*R(IP,KA)+D(2)*R(IP-1,KA))/DT
      ZC=(D(1)*Z(IP,KA)+D(2)*Z(IP-1,KA))/DT
   20 DD(K)= SQRT((RP-RC)**2+(ZP-ZC)**2)
      DT=DD(1)+DD(2)
      AP=(DD(1)*AA+DD(2)*AB)/DT
      RETURN
      END
```

```
      SUBROUTINE SPLINT (X,Y,N,Z,MAX,YINT)
      DIMENSION X(50),Y(50),S(50),A(50),B(50),C(50),F(50),W(50),SB(50),
     1G(50),FM(50),Z(50),YINT(50)
      COMMON Q
      INTEGER Q
      DO 10 I=2,N
      S(I)=X(I)-X(I-1)
   10 CONTINUE
      NO=N-1
      DO 20 I=2,NO
      A(I)=S(I)/6.0
      B(I)=(S(I)+S(I+1))/3.0
      C(I)=S(I+1)/6.0
   20 F(I)=(Y(I+1)-Y(I))/S(I+1)-(Y(I)-Y(I-1))/S(I)
      A(N)=-.5
      B(1)=1.0
      B(N)=1.0
      C(1)=-.5
      F(1)=0.0
      F(N)=0.0
      W(1)=B(1)
      SB(1)=C(1)/W(1)
      G(1)=0.0
      DO 30 I=2,N
      W(I)=B(I)-A(I)*SB(I-1)
      SB(I)=C(I)/W(I)
   30 G(I)=(F(I)-A(I)*G(I-1))/W(I)
      FM(N)=G(N)
      DO 40 I=2,N
      K=N+1-I
   40 FM(K)=G(K)-SB(K)*FM(K+1)
      DO 90 I=1,MAX
      K=2
      IF(Z(I)-X(1)) 60,50,70
   50 YINT(I)=Y(1)
      GO TO 90
   60 IF(Z(I).LT.(1.1*X(1)-.1*X(2)))WRITE (6,1000)Z(I)
      GO TO 85
 1000 FORMAT (17H OUT OF RANGE Z =F10.6)
   65 IF(Z(I).GT.(1.1*X(N)-.1*X(N-1))) WRITE (6,1000)Z(I)
      K=N
      GO TO 85
   70 IF(Z(I)-X(K)) 85,75,80
   75 YINT(I)=Y(K)
      GO TO 90
   80 K=K+1
      IF(K-N) 70,70,65
   85 YINT(I) = FM(K-1)*(X(K)-Z(I))**3/6./S(K)+FM(K)*(Z(I)-X(K-1))**3/6.
     1/S(K)+(Y(K)/S(K)-FM(K)*S(K)/6.)*(Z(I)-X(K-1))+(Y(K-1)/S(K)-FM(K-1)
     2*S(K)/6.)*(X(K)-Z(I))
   90 CONTINUE
      MXA = MAX0(N,MAX)
      IF(Q.EQ.16) WRITE(6,1010) N,MAX,(X(I),Y(I),Z(I),YINT(I),I=1,MXA)
```

```
      SUBROUTINE RBCH(YR,R,Z,T,H,ETA,NOUT)
      DIMENSION YR(6),R(2),Z(2),ETA(2)
      IF( ABS(Z(2)-Z(1)).LT.1.0E-12) GO TO 100
      IF( ABS(R(2)-R(1)).LT.1.0E-12) GO TO 200
      SM=(R(2)-R(1))/(Z(2)-Z(1))
      IF( ABS(YR(6)).LE.1.0E-12) GO TO 10
      IF( ABS(YR(2)).LE.1.0E-12) GO TO 20
      PM=YR(2)/YR(6)
      ZB=(Z(1)*SM-YR(5)*PM+YR(1)-R(1))/(SM-PM)
      RB=SM*(ZB-Z(1))+R(1)
      GO TO 30
   10 ZB=YR(5)
      RB=SM*(ZB-Z(1))+R(1)
      GO TO 30
   20 RB=YR(1)
      ZB=(RB-R(1))/SM+Z(1)
      GO TO 30
  100 IF( ABS(YR(2)).LT.1.0E-12) GO TO 110
      PM=YR(2)/YR(6)
      RB=PM*(Z(1)-YR(5))+YR(1)
      ZB=Z(1)
      GO TO 30
  110 RB=YR(1)
      ZB=Z(1)
      GO TO 30
  200 IF( ABS(YR(6)).LT.1.0E-12) GO TO 250
      PM=YR(2)/YR(6)
      RB=R(1)
      ZB=(RB-YR(1))/PM+YR(5)
      GO TO 30
  250 RB=R(1)
      ZB=YR(5)
   30 DBP= SQRT((RB-YR(1))**2+(ZB-YR(5))**2)
      VM= SQRT(YR(2)**2+YR(6)**2)
      DT=H-DBP/VM
      TB=YR(3)+YR(4)*DBP/VM
      WRITE(6,1000) NOUT,RB,TB,ZB
      GAMMA= ATAN2((R(2)-R(1)),(Z(2)-Z(1))) -1.5707963
      ALPHA=3.1415927+ ATAN2(YR(2),YR(6))
      VN=VM* COS(GAMMA-ALPHA)
      VTR=VM* SIN(GAMMA-ALPHA)
      VNP=-VN*ETA(1)
      VTRP=VTR*ETA(2)
      YR(4)=YR(4)*ETA(2)
      VMP= SQRT(VNP**2+VTRP**2)
      BETA= ATAN2(VTRP,-VNP)
      YR(2)= VMP* SIN(GAMMA+BETA)
      YR(6)= VMP* COS(GAMMA+BETA)
      YR(1)=RB+YR(2)*DT
      YR(3)=TB+YR(4)*DT
      YR(5)=ZB+YR(6)*DT
      T=T+H
      RETURN
```

```
      SUBROUTINE RRBR(YR,THET,Z,T,H,ETA,NOUT,DTHET)
      DIMENSION YR(6),THET(2),Z(2),ETA(2)
      DIMENSION Y(2)
      IF(NOUT.NE.5) GO TO 4
      DO 4 I=1,2
    4 Y(I)=(THET(I)+DTHET)*YR(1)
    5 IF(NOUT.NE.6) GO TO 7
      DO 6 I=1,2
    6 Y(I)=(THET(I)-DTHET)*YR(1)
    7 CONTINUE
      IF(ABS(Z(2)-Z(1)).LT.1.0E-12) GO TO 10
      IF(ABS(Y(2)-Y(1)).LT.1.0E-12) GO TO 20
      SM=(Y(2)-Y(1))/(Z(2)-Z(1))
      IF(ABS(YR(4)).LT.1.0E-12) GO TO 30
      IF(ABS(YR(6 )).LT.1.0E-12) GO TO 40
      PM=YR(4)*YR(1)/YR(6)
      ZB=(YR(3)*YR(1)-PM*YR(5)-Y(1)+SM*Z(1))/(SM-PM)
      YB=SM*(ZB-Z(1))+Y(1)
      GO TO 100
   10 IF(ABS(YR(4)).LT.1.0E-12) GO TO 15
      ZB=Z(1)
      PM=YR(4)*YR(1)/YR(6)
      YB=PM*(Z(1)-YR(5))+YR(3)*YR(3)
      GO TO 100
   15 ZB=Z(1)
      YB=YR(1)*YR(3)
      GO TO 100
   20 IF(ABS(YR(6)).LT.1.0E-12) GO TO 25
      YB=Y(1)
      PM=YR(4)*YR(1)/YR(6)
      ZB=(YB-YR(1)*YR(3)+PM*YR(5))/PM
      GO TO 100
   25 YB=Y(1)
      ZB=YR(5)
      GO TO 100
   30 YB=YR(1)*YR(3)
      ZB=(YB-Y(1)+SM*Z(1))/SM
      GO TO 100
   40 ZB=YR(5)
      YB=SM*(ZB-Z(1))+Y(1)
  100 CONTINUE
      DBP=SQRT((YB-YR(1)*YR(3))**2+(ZB-YR(5))**2)
      VBR=SQRT((YR(4)*YR(1))**2+YR(6)**2)
      DT1=DBP/VBR
      DT=H-DT1
      TB=YB/YR(1)
      RH=YR(2)*DT1+YR(1)
      WRITE(6,1000) NOUT,RB,TB,ZB
      GAMMA= ATAN2((Y(2)-Y(1)),(Z(2)-Z(1)))  -1.5707963
      ALPHA=3.14159 ATAN2((YR(1)*YR(4)),YR(6))
      VN=VBR* COS(GAMMA-ALPHA)
      VT=VBR* SIN(GAMMA-ALPHA)
      VRP=-VN*ETA(1)
```

```
      VTRP=VTR*ETA(2)
      VR(2)=VR(2)*ETA(2)
      VBBP=SQRT(VNP**2+VTRP**2)
      BETA= ATAN2(VTRP,-VNP)
      VR(4)= VBBP* SIN(GAMMA+BETA)/RB
      VR(6)= VBBP* COS(GAMMA+BETA)
      VR(1)=RB+VR(2)*DT
      VR(3)=TB+VR(4)*DT
      VR(5)=ZB+VR(6)*DT
      T=T+H
      RETURN
 1000 FORMAT(29H PARTICLE BOUNCED OFF SURFACE,I5,2X,16H(RB,TB,ZB) ARE (,
     1  3F15.5,1H))
      END
```

The function routine RUNGE has been removed from the published form of this report to protect the copyright of the authors of Reference 5.

```
      SUBROUTINE RNUMBR(RENOLD,DGFC,CD)
      IF( ABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DGFC*0.4
      RETURN
   26 CD=DGFC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=ALOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1  -0.0065*ARE**5)*DGFC
      RETURN
      END
```

```
      SUBROUTINE RADUMP(MX,KMX,MTHTA)
      WRITE(7,100) MX,KMX,MTHTA
  100 FORMAT(3I5)
      RETURN
      END

      SUBROUTINE RAREAD(MX,KMX,MTHTA)
      READ(5,100) MX,KMX,MTHTA
  100 FORMAT(3I5)
      RETURN
      END

      SUBROUTINE RBDUMP(X)
      DIMENSION X(21)
      DO 10 K=1,19,3
   10 WRITE(7,100) X(K),X(K+1),X(K+2),K
  100 FORMAT(3E20.8,12X,2HBB,I3)
      RETURN
      END

      SUBROUTINE RBREAD(X)
      DIMENSION X(21)
      DO 10 K=1,19,3
   10 READ(5,100) X(K),X(K+1),X(K+2)
  100 FORMAT(3E20.8)
      RETURN
      END

      SUBROUTINE RCDUMP(X)
      DIMENSION X(21,21)
      DO 20 I=1,21,1
      DO 10 K=1,19,3
   10 WRITE(7,100) X(I,K),X(I,K+1),X(I,K+2),I,K
   20 CONTINUE
  100 FORMAT(3E20.8,12X,2HRC,2I3)
      RETURN
      END

      SUBROUTINE RCREAD(X)
      DIMENSION X(21,21)
      DO 20 I=1,21,1
      DO 10 K=1,19,3
   10 READ(5,100) X(I,K),X(I,K+1),X(I,K+2)
   20 CONTINUE
  100 FORMAT(3E20.8)
      RETURN
      END

      SUBROUTINE RDDUMP(A,B,C)
      WRITE(7,100) A,B,C
  100 FORMAT(3E20.8)
      RETURN
      END

      SUBROUTINE RDREAD(A,B,C)
      READ(5,100) A,B,C
  100 FORMAT(3E20.8)
      RETURN
      END
```

Example (Part A)

The example presented here uses the ft., lbm., second system of units. The rotor in this case has an inlet radius of 2.961 inches (7.52 cm). The gas flow is that corresponding to a gas with inlet temperature of 2660°R and density of 0.1277 lbm/ft^3.

A more detailed description of the input data format is contained in Reference 9. Only the output from the computer run is included on the following pages.

The solution reveals no large variations in the radius of curvature and smoothly accelerating gas velocities from inlet to exit.

Inputting the variable BCDP as described in Reference 9 with a value of 2 will cause a set of data cards to be punched. These data cards are used as the first part of the input for the particle trajectory program.

RUN NO. 1 INPUT DATA CARD LISTING

```
 10   11   17   13 5390.0000    0.9840   13.0000    1.4000 1715.0000
  0    0    0    0 5942.0000  155.3000    0.1941    0.0010    2.5000
 13    2    2    1    1.0000   -1.0000  -35.0000    1.7500    0.1000
0.00001
 0.3000    0.3400    0.3948    0.4810    0.5412    0.6193    0.7080
 0.8130    0.9100    1.0000
 0.0      -0.0270   -0.0530   -0.0540    0.0090    0.1370    0.4100
 0.7300    0.9100    1.0400
 2.2500    2.0520    1.8610    1.6800    1.6000    1.5341    1.4960
 1.4785    1.4751    1.4750
 2.2500    2.0180    1.7630    1.4120    1.2080    1.0010    0.7790
 0.6811    0.6751    0.6750
 0.0       0.0       0.0      -0.0004   -0.0027   -0.0040   -0.0240
-0.0517   -0.0972   -0.1632   -0.2487   -0.3512   -0.4600
-0.1000    0.0       0.1000    0.2000    0.3000    0.4000    0.5000
 0.6000    0.7000    0.8000    0.9000    1.0000    1.1000
 0.3350    0.3560    0.3220    0.2900    0.2600    0.2330    0.2010
 0.1750    0.1500    0.1280    0.1080    0.0920    0.0790
 0.3750    0.3450    0.3110    0.2800    0.2500    0.2220    0.1910
 0.1650    0.1400    0.1190    0.1000    0.0840    0.0740
 0.3650    0.3330    0.3000    0.2700    0.2400    0.2100    0.1810
 0.1550    0.1310    0.1100    0.0920    0.0780    0.0690
 0.3550    0.3210    0.2900    0.2590    0.2290    0.2000    0.1710
 0.1460    0.1210    0.1010    0.0850    0.0730    0.0640
 0.3450    0.3100    0.2800    0.2490    0.2190    0.1890    0.1610
 0.1360    0.1130    0.0930    0.0790    0.0690    0.0590
 0.3330    0.3000    0.2690    0.2380    0.2080    0.1790    0.1500
 0.1270    0.1040    0.0860    0.0730    0.0630    0.0560
 0.3200    0.2900    0.2580    0.2270    0.1970    0.1660    0.1400
 0.1170    0.0950    0.0760    0.0680    0.0590    0.0520
 0.3100    0.2790    0.2470    0.2150    0.1860    0.1570    0.1320
 0.1370    0.0870    0.0730    0.0620    0.0550    0.0490
 0.3030    0.2680    0.2360    0.2040    0.1750    0.1470    0.1200
 0.0980    0.0790    0.0680    0.0580    0.0510    0.0460
 0.2800    0.2570    0.2240    0.1930    0.1640    0.1380    0.1130
 0.0880    0.0730    0.0620    0.0540    0.0470    0.0430
 0.2510    0.2460    0.2130    0.1820    0.1530    0.1280    0.1020
 0.0790    0.0670    0.0570    0.0500    0.0460    0.0430
 0.2230    0.2230    0.2020    0.1700    0.1410    0.1150    0.0890
 0.0700    0.0610    0.0520    0.4600    0.0430    0.0370
 0.1940    0.1940    0.1900    0.1590    0.1300    0.1020    0.0790
 0.0600    0.0       0.0       0.0       0.0       0.0
 0.1660    0.1660    0.1660    0.1480    0.1180    0.0910    0.0690
 0.0500    0.0       0.0       0.0       0.0       0.0
 0.1370    0.1370    0.1370    0.1370    0.1060    0.0800    0.0590
 0.0400    0.0       0.0       0.0       0.0       0.0
 0.1090    0.1090    0.1090    0.1090    0.0930    0.0700    0.0500
 0.0300    0.0       0.0       0.0       0.0       0.0
 0.0800    0.0600    0.0800    0.0800    0.0800    0.0600    0.0400
 0.0200    0.0       0.0       0.0       0.0       0.0
-0.4000    0.0       0.0000    0.2000    0.3000    0.4000    0.5000
 0.6000    0.7000    0.8000    0.9000    1.0000    1.1000
 0.6500    0.7500    0.8500    0.9500    1.0500    1.1500    1.2500
 1.3500    1.4500    1.5500    1.6500    1.7500    1.8500    1.9500
 2.0500    2.1500    2.2500
```

STAG. SPEED OF SOUND AT INLET = 1192.22

ITERATION NO. 1 MAX. STREAMLINE CHANGE = 0.222071
ITERATION NO. 2 MAX. STREAMLINE CHANGE = 0.208135
ITERATION NO. 3 MAX. STREAMLINE CHANGE = 0.184157
ITERATION NO. 4 MAX. STREAMLINE CHANGE = 0.161830
ITERATION NO. 5 MAX. STREAMLINE CHANGE = 0.141867
ITERATION NO. 6 MAX. STREAMLINE CHANGE = 0.124305
ITERATION NO. 7 MAX. STREAMLINE CHANGE = 0.108968
ITERATION NO. 8 MAX. STREAMLINE CHANGE = 0.095628
ITERATION NO. 9 MAX. STREAMLINE CHANGE = 0.084026
ITERATION NO. 10 MAX. STREAMLINE CHANGE = 0.073936
ITERATION NO. 11 MAX. STREAMLINE CHANGE = 0.065144
ITERATION NO. 12 MAX. STREAMLINE CHANGE = 0.057467
ITERATION NO. 13 MAX. STREAMLINE CHANGE = 0.050730
ITERATION NO. 14 MAX. STREAMLINE CHANGE = 0.044813
ITERATION NO. 15 MAX. STREAMLINE CHANGE = 0.039612
ITERATION NO. 16 MAX. STREAMLINE CHANGE = 0.035022
ITERATION NO. 17 MAX. STREAMLINE CHANGE = 0.030981
ITERATION NO. 18 MAX. STREAMLINE CHANGE = 0.027411
ITERATION NO. 19 MAX. STREAMLINE CHANGE = 0.024260
ITERATION NO. 20 MAX. STREAMLINE CHANGE = 0.021477
ITERATION NO. 21 MAX. STREAMLINE CHANGE = 0.019018
ITERATION NO. 22 MAX. STREAMLINE CHANGE = 0.016839
ITERATION NO. 23 MAX. STREAMLINE CHANGE = 0.014918
ITERATION NO. 24 MAX. STREAMLINE CHANGE = 0.013239
ITERATION NO. 25 MAX. STREAMLINE CHANGE = 0.011765
ITERATION NO. 26 MAX. STREAMLINE CHANGE = 0.010454
ITERATION NO. 27 MAX. STREAMLINE CHANGE = 0.009295
ITERATION NO. 28 MAX. STREAMLINE CHANGE = 0.008263
ITERATION NO. 29 MAX. STREAMLINE CHANGE = 0.007350
ITERATION NO. 30 MAX. STREAMLINE CHANGE = 0.006537
ITERATION NO. 31 MAX. STREAMLINE CHANGE = 0.005815
ITERATION NO. 32 MAX. STREAMLINE CHANGE = 0.005178
ITERATION NO. 33 MAX. STREAMLINE CHANGE = 0.004608
ITERATION NO. 34 MAX. STREAMLINE CHANGE = 0.004102
ITERATION NO. 35 MAX. STREAMLINE CHANGE = 0.003652
ITERATION NO. 36 MAX. STREAMLINE CHANGE = 0.003251
ITERATION NO. 37 MAX. STREAMLINE CHANGE = 0.002896
ITERATION NO. 38 MAX. STREAMLINE CHANGE = 0.002581
ITERATION NO. 39 MAX. STREAMLINE CHANGE = 0.002298
ITERATION NO. 40 MAX. STREAMLINE CHANGE = 0.002048
ITERATION NO. 41 MAX. STREAMLINE CHANGE = 0.001824
ITERATION NO. 42 MAX. STREAMLINE CHANGE = 0.001628
ITERATION NO. 43 MAX. STREAMLINE CHANGE = 0.001449
ITERATION NO. 44 MAX. STREAMLINE CHANGE = 0.001293
ITERATION NO. 45 MAX. STREAMLINE CHANGE = 0.001152
ITERATION NO. 46 MAX. STREAMLINE CHANGE = 0.001026
ITERATION NO. 47 MAX. STREAMLINE CHANGE = 0.000916

DN	Z	R	MA	PRS	WTR	PC
STREAMLINE 1						
0.0	0.0	2.250000	295.632080	28.948502	0.0	0.206545
0.0	-0.027000	2.018000	244.252365	26.055984	0.0	3.418553
0.0	-0.054000	1.786000	224.791565	23.139295	0.0	2.373649
0.0	-0.054000	1.411999	176.907639	20.036743	0.0	17.658295
0.0	0.000000	1.207998	183.239822	18.452895	0.0	10.869154
0.0	0.137000	1.000998	186.012283	17.094681	0.0	14.145462
0.0	0.410000	0.778999	193.714355	15.786360	0.0	11.166971
0.0	0.730000	0.680000	248.374222	14.876602	0.0	14.036795
0.0	0.910000	0.675000	399.253906	13.774944	0.0	-0.119723
0.0	1.039999	0.675000	537.193115	12.531340	0.0	-0.055512
STREAMLINE 3						
0.060683	0.060684	2.250000	296.517578	28.940155	0.0	1.552913
0.080352	0.053011	2.025412	248.089966	26.072495	0.0	3.637551
0.118628	0.063082	1.733403	240.976563	23.196121	0.0	9.314014
0.209881	0.133653	1.506301	236.871826	20.346039	0.0	16.129425
0.256463	0.217105	1.361281	248.646240	19.911536	0.0	15.952235
0.305679	0.342212	1.227825	270.109154	17.807739	0.0	13.123368
0.340734	0.540776	1.093552	309.077148	16.529480	0.0	8.132901
0.315767	0.762647	0.994385	409.825195	15.038394	0.0	5.953679
0.271172	0.910000	0.946182	509.270020	13.836711	0.0	5.602612
0.241898	1.027919	0.916604	596.654053	12.715893	0.0	3.030144
STREAMLINE 5						
0.121951	0.121953	2.250000	298.513184	28.921249	0.0	2.357697
0.157672	0.130002	2.032544	256.038330	26.069077	0.0	5.717117
0.221103	0.162993	1.810263	262.155762	23.215240	0.0	10.274747
0.344822	0.254303	1.586439	285.877197	20.447647	0.0	15.918599
0.406078	0.335959	1.448825	310.122803	19.164154	0.0	16.075500
0.460506	0.445950	1.342490	342.539795	17.975464	0.0	13.792950
0.500131	0.601954	1.245349	399.685547	16.637680	0.0	9.973216
0.484834	0.780124	1.162223	506.184082	14.985494	0.0	7.670432
0.446295	0.910000	1.121312	593.957275	13.717205	0.0	6.933749
0.420351	1.019007	1.094841	655.230469	12.794467	0.0	3.184549
STREAMLINE 7						
0.180830	0.180833	2.250000	301.141846	28.896210	0.0	2.523566
0.231808	0.203823	2.039383	265.339619	26.044357	0.0	6.033831
0.310030	0.249865	1.829280	285.497061	23.183777	0.0	10.567771
0.447203	0.345642	1.612283	335.270996	20.390320	0.0	15.847062
0.511862	0.421132	1.511561	375.252686	19.052689	0.0	17.242813
0.567918	0.510312	1.422442	425.136963	17.756897	0.0	16.933450
0.611972	0.644880	1.344129	496.145264	16.333649	0.0	14.253599
0.613006	0.793069	1.285758	594.472656	14.713057	0.0	11.125129
0.588000	0.910000	1.258023	666.609375	13.561134	0.0	6.804639
0.562545	1.011756	1.239853	711.296387	12.801001	0.0	3.117519
STREAMLINE 9						
0.240468	0.240472	2.250000	304.130371	28.867447	0.0	2.336857
0.302183	0.273896	2.045975	277.515381	26.006424	0.0	5.366113
0.388520	0.326542	1.843051	311.487061	23.096756	0.0	10.363492
0.529047	0.419555	1.643219	389.917480	20.163773	0.0	16.187617
0.594429	0.487371	1.563353	452.623291	18.635101	0.0	20.007431
0.650769	0.573596	1.483581	528.607422	17.065231	0.0	23.723633
0.700519	0.674865	1.425900	605.048340	15.604924	0.0	21.313599
0.712856	0.803703	1.388061	680.498291	14.257373	0.0	14.370979
0.698216	0.910000	1.373243	728.133789	13.408404	0.0	5.760336
0.690256	1.005528	1.364419	761.512207	12.798480	0.0	2.652925
STREAMLINE 11						
0.299995	0.300000	2.250000	307.447510	28.835251	0.0	2.002020
0.368506	0.340000	2.051998	289.208008	25.956879	0.0	6.448373
0.456393	0.394803	1.863998	343.774414	22.947128	0.0	9.891227
0.598570	0.481000	1.679998	453.138672	19.734543	0.0	16.634583

	0.660963	0.541200	1.577998	551.020996	17.784127	0.0	25.383804
	0.718893	0.619300	1.536099	670.338838	15.615330	0.0	38.459375
	0.776429	0.738000	1.495998	727.275635	14.445056	0.0	29.738525
	0.802772	0.813000	1.478498	759.715088	13.734666	0.0	15.735647
	0.800066	0.910000	1.475098	777.707520	13.326412	0.0	2.621365
	0.800969	1.000000	1.474998	804.683594	12.826401	0.0	1.255404

ITERATION NO. 48 MAX. STREAMLINE CHANGE = 0.000819

ITERATION NO. 49

AL	RC	SM	BETA	TT	SA	SB	SC	SD
STREAMLINE 1								
-96.791061	0.266545	0.0	-34.811630	0.080000	-0.016304	4747.835938	0.137425	0.358289
-96.335129	0.418553	0.233566	-13.164053	0.146281	-0.345529	1000.376663	0.403030	27.377579
-94.833146	0.873640	0.489888	-0.021095	0.219230	-0.138752	1494.835938	2.365304	125.762009
-81.547180	17.656295	0.840890	-0.000131	0.289255	2.595669	452.732422	17.466476	-67.138231
-65.289501	10.869164	1.056396	0.033368	0.291358	4.543654	-422.876465	9.873999	193.330963
-50.600610	14.145462	1.297773	-0.071968	0.273430	8.978398	-109.570862	10.930734	117.837570
-28.514648	11.100991	1.649643	-4.242905	0.214962	9.674347	856.984619	5.255947	408.039063
-5.107764	14.036795	1.984607	-22.761505	0.152448	9.225153	3764.851563	0.833668	7254.128906
0.328677	-0.110723	2.164676	-33.027679	0.124522	-5.359133	5879.031250	0.002682	12124.316406
-0.022912	-0.055512	2.294676	-37.600525	0.107411	-6.653266	6574.746094	-0.002661	13236.928125
STREAMLINE 3								
-93.117676	1.552913	0.0	-34.360129	0.080000	-0.056727	4694.648438	1.041507	-3.471580
-90.436035	3.607551	0.224719	-13.826351	0.144130	-0.026160	853.124023	3.480257	-0.351072
-83.693914	9.014018	0.461741	-0.187378	0.202734	0.989301	308.539429	8.959467	-33.639497
-66.571472	16.129425	0.753329	-0.058870	0.218703	6.413112	-340.042236	14.799632	178.280319
-53.512100	15.952236	0.920084	-0.675334	0.208839	9.483495	-849.320557	12.821884	899.535371
-40.637040	13.123368	1.103010	-2.979346	0.187637	9.909909	-363.689453	8.495857	1654.796875
-28.451492	8.134491	1.342655	-14.068773	0.152189	6.079705	1788.532959	3.294352	4383.519531
-20.309250	5.953609	1.585842	-32.615382	0.127046	0.454260	5048.824219	0.168118	9177.093750
-15.628882	6.002612	1.740787	-41.269531	0.113726	-1.925781	6435.250000	-0.538733	10480.882813
-12.377753	3.033144	1.862358	-45.235260	0.103818	-5.134829	7030.019531	-1.173935	10658.195313
STREAMLINE 5								
-89.346924	2.357699	0.0	-34.999130	0.080000	0.017973	4510.097656	1.576772	6.821116
-85.310944	5.717117	0.217644	-11.395299	0.141123	0.448849	471.819824	5.472179	54.945159
-76.994675	13.274740	0.442554	-0.967580	0.174996	2.311883	-678.640625	10.009554	245.971451
-60.754181	15.916599	0.702721	-1.036364	0.175554	7.772088	-1268.121094	13.880455	1187.130586
-49.684494	16.075500	0.845901	-2.810857	0.165531	10.355851	-1159.979980	12.204514	2174.252930
-38.660919	13.752950	0.998938	-8.611643	0.148094	10.328422	213.088379	8.263069	3841.041016
-28.082184	9.970216	1.185381	-21.618515	0.128791	6.289762	3145.394531	3.355909	7227.546875
-19.899139	7.670432	1.379328	-38.252792	0.115591	0.489895	6314.210938	0.177331	10574.656250
-15.149429	6.933749	1.515994	-46.289529	0.108504	-2.366899	7585.464844	-0.640830	9550.992188
-12.483317	3.164549	1.628169	-50.391349	0.103238	-5.169444	8142.582031	-1.144461	7938.335938
STREAMLINE 7								
-85.563324	2.323566	0.0	-34.968750	0.080011	0.127869	4211.714844	1.648020	72.231430
-81.395660	6.093831	0.211868	-12.066625	0.137025	0.854680	31.317627	5.648438	288.362988
-73.105225	10.907771	0.426956	-1.469376	0.146942	3.067115	-1609.209717	10.098437	915.695313
-53.133614	15.847062	0.664221	-2.822482	0.144654	8.521630	-2249.380127	13.400957	2750.246582
-48.146920	17.242813	0.789701	-6.852255	0.136819	11.228038	-1505.800781	12.534456	4618.347656
-37.309860	16.833450	0.921819	-14.509953	0.124902	12.018642	902.051514	9.159025	6884.910156
-26.175766	14.253599	1.079753	-28.155487	0.114753	7.955796	4713.238281	3.910559	9694.574219
-16.549713	11.123129	1.229660	-42.659550	0.107173	1.484909	7437.253125	0.441253	10342.351563
-11.476128	6.804609	1.350070	-49.896286	0.104682	-2.813398	8357.382813	-0.571172	8024.253125
-9.063053	3.117519	1.453934	-53.557333	0.099668	-5.198359	8771.852031	-0.829669	5839.219938
STREAMLINE 9								
-82.304672	2.330857	0.0	-34.990158	0.080081	0.194861	3877.222412	1.442108	198.372571
-73.601242	5.366113	0.206844	-12.969493	0.114779	0.992231	-462.827637	4.920492	653.158691
-71.359677	10.363492	0.413476	-2.781227	0.123475	3.346933	-2642.400635	9.753119	1845.810986
-57.334656	16.187607	0.631187	-5.975796	0.121622	8.563377	-3490.194580	13.356611	5398.914063
-47.626007	20.007431	0.742376	-11.343925	0.117098	12.665088	-2411.885986	13.882699	8586.536719
-35.386230	23.722653	0.858424	-20.683314	0.110698	15.919262	2309.855957	11.307494	10965.921875
-22.290340	21.303009	0.973460	-34.048294	0.105247	10.893934	6522.996094	4.465796	10803.135469
-11.312394	14.370979	1.108619	-46.285843	0.103303	2.216461	8268.464844	0.443393	7934.511719
-6.344591	5.760336	1.216086	-52.737564	0.100098	-3.436270	8768.257813	-0.382075	5619.281250

-4.467793	2.652925	1.312021	-56.309677	0.077732	-5.274943	9051.046875	-0.410312	4511.976594
STREAMLINE 11								
-79.737638	2.302020	3.0	-35.144241	0.383277	0.197941	3641.653564	1.112089	355.648438
-76.358673	4.446370	0.202031	-14.161962	0.095960	0.915480	-1085.472412	3.921233	1195.637451
-70.267807	9.691227	0.400707	-4.922491	0.103803	3.282253	-3718.677666	9.157797	3377.983398
-57.715134	16.634583	0.601185	-9.341342	0.105953	8.462729	-5657.800781	13.394559	9199.488281
-47.657852	25.363804	0.701305	-15.723735	0.104203	15.290913	-4649.562031	16.779678	15753.722656
-31.560181	38.455576	0.803493	-27.270691	0.101182	24.249039	4225.871094	14.894911	15374.472344
-15.618239	29.736525	0.900328	-39.563538	0.099291	13.767314	8392.257813	3.848630	7444.870625
-4.485056	15.735647	1.006476	-49.197632	0.099745	2.047854	8514.765625	0.160632	3137.242188
-0.525846	2.621365	1.103536	-54.857422	0.097805	-4.571162	8839.652344	-0.041954	2781.229248
0.393810	1.455404	1.193536	-58.255966	0.095035	-5.536105	9161.582031	0.029017	4238.385938

ON	Z	R	WA	PPS	WTP	PC
STREAMLINE 1						
0.0	0.0	2.250000	295.632380	28.948502	-22.249878	1.274565
0.0	-0.027000	2.013333	244.249451	26.056015	-232.965494	0.418557
0.0	-0.053000	1.763000	224.791565	23.130295	-306.866699	2.373660
0.0	-0.054000	1.411999	176.912857	20.036713	-117.234940	17.658295
0.0	-0.039000	1.237998	183.248199	18.452859	25.653763	13.367154
0.0	0.137000	1.033998	186.016708	17.094650	115.757858	14.145462
0.0	0.410000	0.778999	153.728287	15.796307	197.654694	11.166991
0.0	0.733000	0.633000	248.374222	14.976602	232.854397	14.006795
0.0	0.910000	0.675000	399.277832	13.774780	344.622803	-0.110723
0.0	1.034999	0.675000	537.226807	12.531041	458.083984	-0.055512
STREAMLINE 3						
0.060684	0.060685	2.250000	296.518066	28.940155	-23.380908	1.553555
0.080363	0.053022	2.025413	248.088745	26.072510	-221.903809	3.609227
0.116055	0.063108	1.783408	240.981201	23.196152	-217.159988	9.214427
0.209931	0.133698	1.506022	236.892212	20.346100	-3.155060	16.130051
0.258524	0.217154	1.361318	248.677002	19.011597	97.359497	15.951024
0.305947	0.342257	1.227876	270.160137	17.807861	177.968536	13.122724
0.349863	0.541803	1.093717	309.125000	16.529510	200.767746	8.130458
0.315625	0.762653	0.994143	409.868408	15.038340	283.413574	5.951674
0.271216	0.910000	0.943227	509.308594	13.806561	431.695068	6.673375
0.241932	1.027917	0.916639	596.692627	12.715644	499.987305	3.030512
STREAMLINE 5						
0.120953	0.120955	2.250000	298.514404	28.921249	-53.446077	2.358137
0.157689	0.137019	2.032345	256.043283	26.069077	-156.200607	5.718139
0.221138	0.163028	1.810275	262.165283	23.215225	-119.649353	10.275573
0.340881	0.234356	1.566464	285.907715	20.447632	45.509430	15.917777
0.406146	0.336014	1.448866	310.162842	19.164154	127.858746	16.073807
0.460579	0.445999	1.342544	342.588867	17.975464	170.034058	13.792167
0.503204	0.601983	1.240916	399.747070	16.637604	190.386154	9.969821
0.484874	0.780132	1.162292	506.234863	14.985426	318.764404	7.668449
0.446358	0.910000	1.121375	594.001221	13.717064	464.719971	6.928842
0.420404	1.019005	1.094894	655.275879	12.794203	561.632324	5.183792
STREAMLINE 7						
0.180832	0.180835	2.250000	301.143799	28.896210	-83.179711	2.523822
0.231824	0.205839	2.037304	266.363770	26.044327	-74.280319	6.004385
0.310063	0.299898	1.829287	285.500000	23.183716	-33.257858	13.569247
0.447654	0.345688	1.612316	335.303467	20.390213	48.338348	15.845944
0.511519	0.421174	1.511596	375.299072	19.052551	112.395637	17.242065
0.567979	0.518053	1.422188	425.196289	17.756668	143.375839	16.833572
0.612034	0.644904	1.344188	476.208984	16.333389	195.629913	14.253938
0.610069	0.793076	1.286421	594.514648	14.712975	374.632813	11.122614
0.563056	0.910000	1.253381	666.639404	13.561130	532.299316	6.801544
0.562596	1.011753	1.239911	711.337891	12.803856	625.729736	3.116441
STREAMLINE 9						
0.240469	0.240474	2.250000	304.132813	28.867447	31.448715	2.337012
0.306191	0.273907	2.046875	277.520752	26.006363	-19.822937	5.366443
0.388540	0.322562	1.846355	311.530732	23.096664	-21.733327	13.383789
0.529678	0.419582	1.644232	389.960918	20.163666	40.653183	16.187210
0.594164	0.487399	1.540373	452.657715	18.634903	83.171738	15.335753
0.653856	0.573621	1.433603	528.653076	17.064893	106.458679	13.726624
0.700557	0.670880	1.325934	605.083984	15.604748	232.032059	21.301834
0.712893	0.803706	1.300098	680.514648	14.257491	466.917725	14.377267
0.696251	0.910000	1.373279	729.145625	13.409529	607.798496	5.759339
0.690207	1.005527	1.364453	761.541504	12.798284	680.014160	2.652539
STREAMLINE 11						
0.259994	0.259995	2.250000	317.450195	28.835220	138.615967	2.032920
0.368566	0.340000	2.051998	289.211426	25.956833	26.414413	4.448370
0.456393	0.394800	1.860998	340.782715	22.947037	-24.681442	9.691227
0.590370	0.481000	1.679998	453.139859	19.734634	19.779663	16.634583

0.660983	0.541200	1.[illegible]	551.003174	17.784256	-13.863796	25.383804
0.716693	0.619300	1.536037	670.272217	15.613315	77.112167	38.459976
0.776428	0.700000	1.[illegible]	727.249347	14.445449	359.711670	29.738525
0.802712	0.813000	1.476643	759.687500	13.735049	597.143311	15.735647
0.800066	0.910000	1.475098	777.696777	13.326551	680.907959	2.621365
0.800968	1.000000	1.474998	804.698975	12.826185	718.491123	1.255434

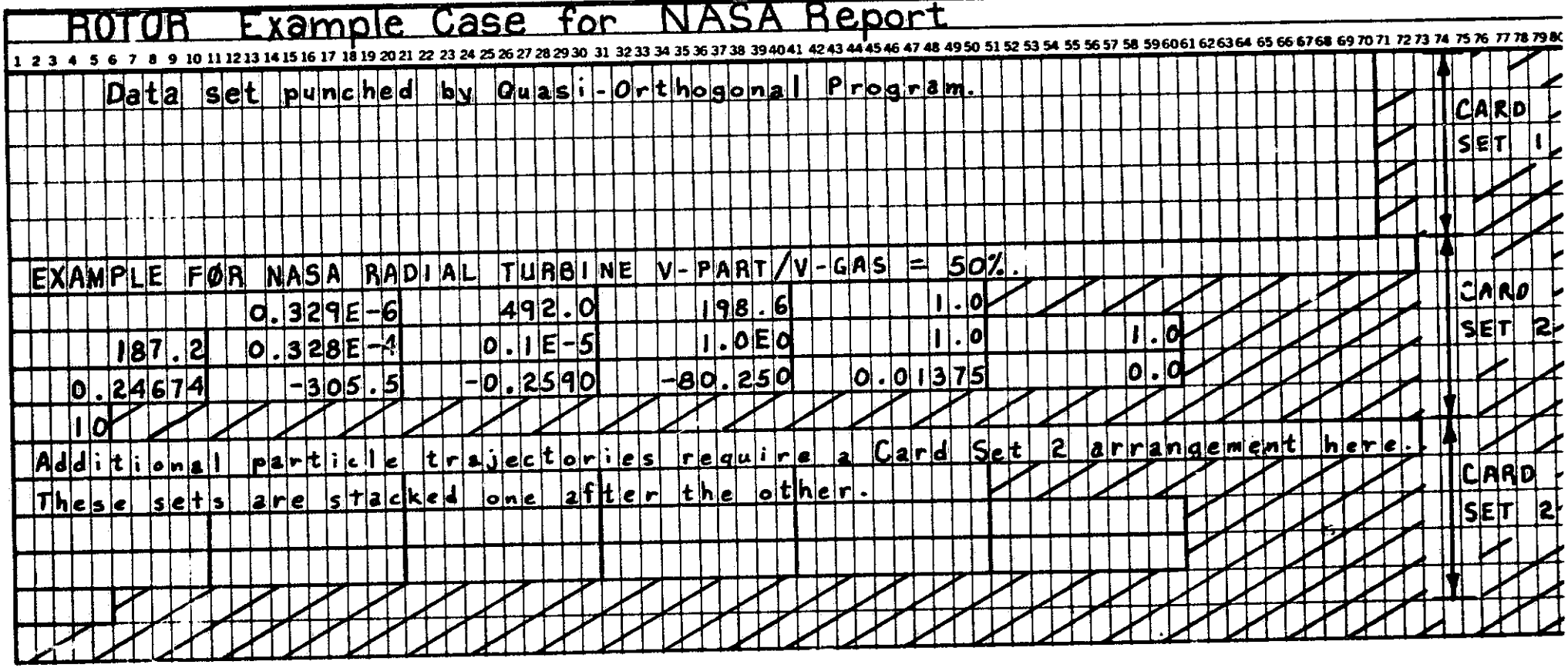
Example (Part B)

The example presented here uses the ft., lbm., second system of units. The rotor in this case has an inlet radius of 0.26475 ft. (7.52 cm). The gas flow is that corresponding to a gas with inlet temperature of 2660°R and density of 0.1277 lb/ft^3.

The particle studied has specific gravity of 2 and a diameter corresponding to about 10 microns. The velocity is initially in the same direction as the gas flow solution with a magnitude of one half the gas velocity. The particle bounces off the shroud surface before passing through the turbine.

The following pages contain a computer code sheet with the data arranged in the proper columns, and the output for this example. The block indicated as that punced by the Quasi-Orthogonal program is approximately a box of cards that is required to transfer the Quasi-Orthogonal information to this program.

ROTOR Example Case for NASA Report



IMPORTANT DATA FROM QUASI-CRTH-CODAAL PROGRAM

NX	NNX	NTHTA
10	21	10

GAMMA	TEMP	RHC	RGAS	SFACT	ZSPLIT	h
1.3900	2460.0000	0.1277	1715.0000	1.0000	-0.0833	7930.0000

NR	NTH	PLOSS
12.0000	484.0000	855.3596

HUB COORDINATES		TIP COORDINATES	
R	Z	R	Z
0.244750	0.0	0.244750	0.026483
0.227942	0.000167	0.229500	0.027483
0.211000	0.000833	0.212400	0.029583
0.194642	0.002500	0.195583	0.036400
0.178117	0.005833	0.182442	0.045817
0.170958	0.010167	0.170950	0.059150
0.162500	0.041667	0.162500	0.075500
0.072750	0.065167	0.157500	0.093500
0.044058	0.089833	0.156150	0.110233
0.042500	0.125000	0.155833	0.125000

X1 ARRAY					
0.0	0.01250	0.02500	0.03500	0.05000	0.06500
0.08000	0.10000	0.11000	0.12500		

THETA ARRAY					
0.0	0.0	0.0	-0.00066	-0.00741	-0.02760
-0.07070	-0.17720	-0.25160	-0.38320		

EXAMPLE FOR NASA RADIAL TURBINE V-PART/V-GAS = SC9.

VISREF	TREF	TSLT	DGFC
0.42960F-06	492.0000	198.6000	1.0000

RMFP	FIAP	P	TMAX	ETA-N	ETA-T
2.0000	0.224F-04	0.100F-05	1.0000	1.0000	1.0000

YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)
0.24674	-305.50000	-0.25500	-80.25000	0.01375	0.0

PRINT DATA EVERY 10 STEP(S)

STEP	T	YR(1)	YR(2)	YR(3)	YR(4)	YR(5)	YR(6)	RENOLD
0	0.0	0.24674	-305.50000	-0.25500	-80.25000	0.01375	0.0	
10	0.999999E-05	0.24244	-472.29395	-0.26650	-158.01053	0.01512	197.88184	568.81372
PARTICLE BOUNCED OFF SURFACE	6 (RR,TR,ZR) ARE (			0.23868	-0.26180	0.016291		
20	0.199999E-04	0.23737	-523.75562	-0.26144	91.87747	0.01657	104.42526	675.89355
30	0.299997E-04	0.23215	-535.37500	-0.26166	-105.81550	0.01736	58.58961	532.99731
PARTICLE BOUNCED OFF SURFACE	6 (RR,TR,ZR) ARE (			0.23160	-0.26180	0.017431		
PARTICLE BOUNCED OFF SURFACE	6 (RR,TR,ZR) ARE (			0.22725	-0.26180	0.017861		
40	0.399995E-04	0.22706	-500.74372	-0.26176	95.76056	0.01782	35.60748	534.09888
50	0.499993E-04	0.22151	-554.41016	-0.26075	119.60564	0.01858	100.59963	545.00684
60	0.599991E-04	0.21569	-586.73975	-0.25940	140.53916	0.01962	132.96960	513.66211
70	0.699988E-04	0.20977	-602.44141	-0.25793	158.59013	0.02070	153.81693	629.21680
80	0.799986E-04	0.20387	-612.23730	-0.25520	333.42663	0.02261	210.18492	504.37915
90	0.899984E-04	0.19753	-616.78198	-0.25156	375.06860	0.02472	205.51757	522.54712
100	0.999982E-04	0.19141	-598.64209	-0.24771	419.00806	0.02700	251.27173	522.74585
110	0.109998E-03	0.18556	-580.15259	-0.24304	505.07764	0.03024	344.62891	489.57739
120	0.119998E-03	0.17970	-554.71118	-0.23775	540.29297	0.03265	338.19409	528.46460
130	0.129998E-03	0.17428	-486.02441	-0.23411	301.82593	0.03799	487.57153	420.81763
140	0.139997E-03	0.16970	-460.05347	-0.23108	302.96655	0.04308	523.16162	458.98584
150	0.149997E-03	0.16500	-485.40381	-0.22802	325.90161	0.04837	527.48901	495.85156
160	0.159997E-03	0.16033	-482.67188	-0.22795	-535.23145	0.05393	622.55811	537.72119
170	0.169997E-03	0.15569	-349.56396	-0.23446	-684.70874	0.06042	665.54932	287.07422
180	0.179996E-03	0.15318	-345.52285	-0.24094	-585.80200	0.06713	672.39111	348.02832
190	0.189996E-03	0.14950	-278.50894	-0.25277	-2210.46854	0.07403	729.54468	564.73730
200	0.199996E-03	0.14720	-248.72145	-0.27708	-2512.53174	0.08148	754.64673	172.18631
210	0.209996E-03	0.14484	-248.28708	-0.30157	-2338.92529	0.08905	757.38306	236.39944
220	0.219996E-03	0.14272	-182.62236	-0.33764	-4399.33203	0.09886	790.50073	374.36108
230	0.229995E-03	0.14065	-175.25328	-0.38300	-4575.14063	0.10478	797.69580	73.77003
240	0.239995E-03	0.13931	-137.42236	-0.43256	-5745.90234	0.11274	801.88745	471.13086
250	0.249994E-03	0.13804	-121.78424	-0.49334	-6215.82422	0.12075	806.32080	159.53368
255	0.254994E-03	0.13721	-121.57803	-0.53087	-6276.99609	0.12563	807.72144	63.94197

1 255

VANPY - Particles in a Turning Vortex

This program calculates the trajectory of a particle in an inward turning vortex after first calculating the flow through such geometry.

The fluid flow solution is based on the principal of constant, $\lambda = V_u r$ along the streamlines, although the values λ, may be different for different streamlines. The variation in the radius of curvature of the streamlines between hub and shroud is taken to be linear. The original program was supplied by NASA Lewis Research Center, but was modified such that the input of data is easier and the velocity signs are consistent with those used in the solution of the particle trajectories.

Method

Figure 24 is a flow diagram of the program VANPY. The results of the fluid solution are stored in arrays that specify the velocity vectors of the gas at the intersection points of the orthogonals and streamlines. With the fluid solution completed, the program uses a Runge-Kutta technique to integrate the three-dimensional equations of motion of the particle.

The integration of the equations of motion over one time increment is first carried out using the gas velocity and the properties of the gas at point A to determine the new particle location B. The program then calls BLOCAT and POLATE to determine the gas velocity components at B_1. A corrected gas velocity components are calculated from B_1 with the mean values at A and B_1. The program then integrates the equations of motion again starting from A to find the corrected particle location B_2. BLOCAT and POLATE are called again to give the gas velocity components at B_2, and these velocity components are compared to the corresponding values at B_1, if the difference is large, the previous iteration is repeated. Once the iteration has converged, the trajectory to point B has been determined and this point is used as the initial point for the next time increment.

The subroutine CONTIN is used to iterate for the hub surface velocity, the iteration technique is based on the mass flow calculated from the fluid solution.

The subroutine PABC calculates the coefficients of the parabola $y = ax^2 + bx + c$, passing through three given points. This subroutine is used in the fluid solution.

The subroutine FNTGRL performs the integration of a function F(1) over equal size increments. This subroutine is used in the fluid solution.

The subroutine SPLINT interpolates points along a spline fit curve that lie between the points specifying the curve.

The subroutine RNUMBR is used to determine the drag coefficient based on a curve fit of the drag versus Reynolds Number data. Equations 20 and Figure 3 demonstrate the fit of these equations to the data.

The subroutine RUNGE uses a fourth order method to integrate a system of simultaneous first order ordinary differential equations across one time step. Reference 5 explains this subroutine in more detail.

The subroutine BLOCAT is used to determine the subscripts of the grid points that surround the location of the particle. Figure 15 shows the particle within a typical set of orthogonals and streamlines. The subroutine returns the values of IP and KP that locate the particle within a particular grid. If the particle is no longer within the boundaries of the flow field, the subroutine returns NOUT, which is a code specifying where the particle has moved out of bounds.

The subroutine POLATE interpolates the value of a variable whose values are specified at the four grid points surrounding the particle. Referring to Figure 15, the subroutine first determines the distances D(1) and D(2), and based on these distances, determines a weighted average of the variable at two locations on adjacent streamlines. These values are AA and AB. Then the subroutine determines DD(1) and DD(2) and uses these distances to get the weighted average of AA and AB at the position occupied by the particle.

The subroutine RBCH is used when the particle rebounds from the case or the hub. It is called whenever the particle position is outside the case or hub boundaries. The subroutine returns to the previous position, and linearly extends the trajectory over one time increment, with the bounce occurring at some portion of the time segment. The subroutine writes "BOUNCE OFF SURFACE (NOUT) ..." and prints the location of the bounce.

Input

The input cards take the following format. Any consistent system of units can be used. An example of the input data is included with the example case presented after the program listing. The data cards cover two main groups;

Group A

MX, KMX, Z1, Z2, R1, R2	(2I5, 4F10.5)
GAMMA, CP, TIP, RHOIP, WRFL	(5F10.5)
RC(2, 1) ... RC(2, KMX)	(8F10.5)
AL(1, 1) ... AL(MX, 1)	(8F10.5)
LAMBDA(1, 1) ... LAMBOA (1, KMX)	(8F10.5)

Group B

TITLE	(18A4)
VISREF, TREF, TSUT, DGFC	(D20.5, 3F10.3)
RHOP, DIAP, H, TMAX, ETA-N, ETA-T	(F10.2, 3D10.4, 2F10.3)
YR(1), YR(2), YR(3), YR(4), YR(5), YR(6)	(6F10.3)
NSTEP	(I5)

Data Group A.

These cards specify the parameters of the gas flow solution. If only these input cards were fed in the input, the fluid solution only will be obtained. Figure 25 indicates with more clarity some of the terms listed below.

MX - Number of orthogonals.

KMX - Number of streamlines.

Z1 - Axial location of first orthogonal. (Length)

Z2 - Axial location of the circle center. (Length)

R1 - Radial location of last orthogonal. (Length)

R2 - Radial location of the circle center. (Length)

GAMMA - Ratio of specific heats.

CP - Specific heat at constant pressure. (C_f g) (Length^2/Time^2 - Degrees Abs)

TIP - Inlet stagnation temperature. (Degrees Abs.)

RHOIP - Inlet stagnation density. (Mass/Length^3)

WTFL - Fluid mass flow. (Mass/Time)

RC Array - Radius of curvature of streamlines 1 through KMX at orthogonals 2 through (MX-1). (Length). The radius of curvature of the streamlines along orthogonal 1 and MX are set equal to 10,000 by the program.

AL Array - The angle α corresponding to the flow directions at the orthogonals 1 through MX, this angle is negative in the direction indicated in Figure 25. (Degrees)

LAMBDA Array - The term $\lambda = rV_u$ for each streamline from 1 through KMX. (Length^2/Time)

Data Group B.

These cards specify the variables that are used in determining the trajectory. For studies involving more than one particle, sets of input cards for different particles can be stacked together.

TITLE - The first card is reproduced at the top of the first page of output for each particle. Any statement in columns 2 through 72 will be reproduced.

VISREF - Reference viscosity corresponding to TREF. Used in Sutherland's Law. (Mass/Length Time)

TREF - Reference temperature corresponding to VISREF. Used in Sutherland's Law. (Degrees Abs.)

TSUT - Constant used in Sutherland's Law. Either 198.6°R or 110.0°K.

DGFC - Drag factor. The drag coefficient based on Reynold's Number is multiplied by DGFC. Except in very special cases, this should be 1.0.

RHOP - Particle density. (Mass/Length^3)

DIAP - Particle diameter. (Length)

H - Time increment used in integration process. (Time)

TMAX - Program stops if time exceeds TMAX. (Time)

ETA-N - Normal restitution coefficient.

ETA-T - Tangential restitution coefficient.

YR(1) - Initial radial position of the particle. (Length)

YR(2) - Initial radial velocity component of the particle. (Length/Time)

YR(3) - Initial angular position of the particle.

YR(4) - Initial angular velocity of the particle. (Time^{-1})

YR(5) - Initial axial position of the particle. (Length)

YR(6) - Initial axial velocity of the particle. (Length/Time)

NSTEP - The program prints out trajectory information every NSTEP time increments.

Output

An example listing of typical output is included after the program listing. This output is divided into several output groups.

Output Group A.

This set of output is an echo check of the input data and includes the calculated critical velocity of the gas. Such data checks are useful in correcting key punch mistakes on the data cards. The variables listed in this group were explained under Input.

Output Group B.

This set of output concerns the solution of the gas flow through the turning vortex. A new page is used for the printed output of different orthogonals. The R and Z arrays locate the intersection points of the orthogonals and the streamlines. The RC, ALPHA and LAMBDA arrays are explained under Input. The rest of the output formats are explained below.

WTFLES - Estimated weight flow when iteration procedure stops. (Mass/Time)

BETA Array - Angle between the meridional velocity component and the velocity vector, at the intersection of the orthogonals and streamlines. (Degrees)

V/V_{cr} Array - Dimensionless speed at the intersection point of the orthogonals and streamlines.

Output Group C.

This set of output corresponds to the particle trajectories. The first part is an echo check of the input cards corresponding to Data Set B. After initializing the variables, the program calculates and prints several similarity parameters to relate particles having similar trajectories. The variables that are printed are explained below.

DELTA - The characteristic length based on the critical density. (Length)

TAU - The particle time constant based on the viscosity at critical flow conditions. (Time)

RECR - The Reynolds Number of a particle whose velocity is equal to half the gas critical velocity.

After the similarity parameters are printed, trajectory information is printed every NSTEP time increments. This information includes the following terms.

L - Number count of the time increments.

T - Time

YR(1) ... YR(6) - Position and velocity components of the particles.

RENOLD - Reynold's Number of the particle at this location.

When the particle passes out of the flow field through the inlet or exit, the program always prints the trajectory information for the last point, which should be just outside the boundaries. Following this the value of the number of iterations, M, is printed. If the iteration procedure for the particle location B has not converged, the value of M will be 101, and a smaller time increment is recommended.

```
C
C PARTICLE THROUGH A TURNING VORTEX          720626
C      THIS PROGRAM CALCULATES THE PARTICLE TRAJECTORY IN AN INWARD
C TURNING VORTEX, AFTER FIRST CALCULATING THE FLOW THROUGH SUCH A
C GEOMETRY.  THE FLUIDS SOLUTION IS BASED ON A STREAMLINE CURVATURE
C METHOD IN WHICH A LINEAR VARIATION IN THE RADUIS OF CURVATURE OF
C THE STREAMLINES IS ASSUMED BETWEEN THE HUB AND THE SHROUD.  THE HUB
C AND THE SHROUD CONTOURS ARE CIRCULAR AND THUS HAVE CONSTANT RADIUS
C OF CURVATURE.  SEE THE PROGRAM DESCRIPTION FOR THE INPUT DATA RELAT-
C ING TO THE FLUID SOLUTION.
C      FROM THE INITIAL POINT, GIVEN AS  INPUT, THE PROGRAM USES A
C RUNGA-KUTTA METHOD TO INTEGRATE THE PARTICLE EQUATIONS OF MOTION TO
C DETERMINE THE LOCATION OF THE PARTICLE BASED ON A GIVEN TIME STEP.
C THE FLUIDS SOLUTION, AT GIVEN GRID POINTS IS USED TO CALCULATE THE
C AERODYNAMIC DRAG ON THE PARTICLE.  SEE THE PROGRAM DESCRIPTION FOR
C THE INPUT DATA AND FORMAT RELATING TO THE TRAJECTORY SOLUTION.
C                                        W. B. CLEVENGER, JR.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 LAMBDA
      INTEGER RUNGE
      DIMENSION V(21,21),BETA(21,21),R(21,21),RC(21,21),AL(21,21),
     1  ANS(21   ),F(21,21),LAMBDA(21,21),RD(21),Z(21,21)
      DIMENSION F1(21)
      DIMENSION STATE(18),YR(6),YRS(6),VR(4),VU(4),VZ(4),RHOA(4),
     1  TEMPA(4),VISTAR(4),ETA(4)
        DIMENSION FR(6),RA(2),ZA(2)
      PI=3.14159
      CONV=PI/180.
      JZ=1
      READ(5,1000) MX,KMX,Z1,Z2,R1,R2
      WRITE(6,2130) MX,KMX,Z1,Z2,R1,R2
      READ(5,1005) GAMMA,CP,TIP,RHOIP,WTFL
      WRITE(6,2140) GAMMA,CP,TIP,RHOIP,WTFL
      READ(5,1010)       (RC(2,K),K=1,KMX)
      WRITE(6,2170) KMX,(RC(2,K),K=1,KMX)
      READ(5,1010)       (AL(I,1),I=1,MX)
      WRITE(6,2160) MX, (AL(I,1),I=1,MX)
      READ(5,1010)       (LAMBDA(1,K),K=1,KMX)
      WRITE(6,2150)KMX, (LAMBDA(1,K),K=1,KMX)
      DO 6 K=1,KMX
      DO 6 I=1,MX
      IF(I.EQ.1) RC(I,K)=-10000.
      IF(I.EQ.MX) RC(I,K)=-10000.
      IF((I.GT.1).AND.(I.LT.MX)) RC(I,K)=RC(2,K)
      AL(I,K)=AL(I,1)
      LAMBDA(I,K)=LAMBDA(1,K)
      IF(I.EQ.1) R(I,K)=R2-RC(2,K)
      IF(I.EQ.MX) R(I,K)=R1
      IF((I.GT.1).AND.(I.LT.MX)) R(I,K)=R2-RC(2,K)*DCOS(AL(I,K)*CONV)
      IF(I.EQ.1) Z(I,K)=Z1
      IF(I.EQ.MX) Z(I,K)=Z2-RC(2,K)
      IF((I.GT.1).AND.(I.LT.MX)) Z(I,K)=Z2+RC(2,K)*DSIN(AL(I,K)*CONV)
```

```
    6 CONTINUE
      DO 100 I=1,MX
      WRITE(6,2060) I
      WRITE(6,2010)
      WRITE(6,2000) (R(I,K),K=1,KMX)
      WRITE(6,2040)
      WRITE(6,2000) (Z(I,K),K=1,KMX)
      WRITE(6,2020)
      WRITE(6,2000)(RC(I,K),K=1,KMX)
      WRITE(6,2030)
      WRITE(6,2000)(AL(I,K),K=1,KMX)
      WRITE(6,2050)
      WRITE(6,2000)(LAMBDA(I,K),K=1,KMX)
      XLENTH=DSQRT((R(I,KMX)-R(I,1))**2+(Z(I,KMX)-Z(I,1))**2)
      VCR=DSQRT(2.0*CP*TIP*(GAMMA-1.0)/(GAMMA+1.0))
      IND=1
      NN=KMX-1
      DELTAN=XLENTH/DFLOAT(KMX-1)
      VM=WTFL/RHOIP/XLENTH/PI/(R(I,1)+R(I,KMX))
      VEST=DSQRT(VM**2+(LAMBDA(I,1)/R(I,1))**2)
      DELTAX=VEST/20.
   10 V(I,1)=VEST
      IF((V(I,1)**2).GT.2.0*CP*TIP) GO TO 45
      RHO=RHOIP*(1.-(V(I,1)**2)/(2.*CP*TIP))**(1./(GAMMA-1.))
      F(I,1)=RHO*VM*R(I,1)
      VA=LAMBDA(I,1)/R(I,1)
      A=1./RC(I,1)
      B=-(VA**2)*(A+DCOS(AL(I,1)*CONV)/R(I,1))
      DO 20 K=1,NN
      V1STAR=V(I,K)+(A*V(I,K)+B/V(I,K))*DELTAN
      A=1./RC(I,K+1)
      VA=LAMBDA(I,K+1)/R(I,K+1)
      B=-(VA**2)*(A+DCOS(AL(I,K+1)*CONV)/R(I,K+1))
      V2STAR=V(I,K)+(A*V1STAR+B/V1STAR)*DELTAN
      V(I,K+1)=(V1STAR+V2STAR)/2.
      VM2=V(I,K+1)**2-VA**2
      VM=0.0
      IF(VM2.GT.0.0) VM=DSQRT(VM2)
      IF((V(I,K+1)**2).GT.2.0*CP*TIP) GO TO 46
      RHO=RHOIP*(1.-(V(I,K+1)**2)/(2.*CP*TIP))**(1./(GAMMA-1.))
   20 F(I,K+1)=RHO*VM*R(I,K+1)
      DO 30 K=1,KMX
   30 F1(K)=F(I,K)
      CALL FNTGRL(DELTAN,F1,KMX,ANS)
      WTFLES=ANS(KMX)*2.*PI
      IF(DABS(WTFLES-WTFL).LE..00001 *WTFL) GO TO 50
      CALL CONTIN(VEST,WTFLES,IND,JZ,WTFL,DELTAX)
      IF(IND.LT.10) GO TO 10
      IF(IND.NE.10)GO TO 40
      WRITE(6,2110) WTFLES
      GO TO 50
   40 WRITE(6,2070)
      GO TO 100
```

```
   45 WRITE(6,2080) V(I,1)
      GO TO 100
   46 WRITE(6,2081) V(I,K+1)
      GO TO 100
   50 DO 60 K=1,KMX
      BETA(I,K)=DARSIN(LAMBDA(I,K)/V(I,K)/R(I,K))/CONV
   60 V(I,K)=V(I,K)/VCR
      WRITE(6,2120) WTFLES
      WRITE(6,2090) (BETA(I,K),K=1,KMX)
      WRITE(6,2100) (V(I,K),K=1,KMX)
  100 CONTINUE
  105 READ(5,3000) (STATE(I),I=1,18)
      WRITE(6,4000)(STATE(I),I=1,18)
      READ(5,3010) VISREF,TREF,TSUT,RAIR,DGFC
      WRITE(6,4010) VISREF,TREF,TSUT,RAIR,DGFC
      READ(5,3020)  RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      WRITE(6,4020) RHOP,DIAP,H,TMAX,ETA(1),ETA(2)
      T=0.0
      READ(5,3030) (YR(I),I=1,6)
      WRITE(6,4030)(YR(I),I=1,6)
      READ(5,3080) NSTEP
      WRITE(6,4080) NSTEP
      RHOCR=RHOIP*(2.0/(GAMMA+1.0))**(1.0/(GAMMA-1.0))
      DELTA=RHOP*DIAP/RHOCR/0.3
      TCR=(2.0/(GAMMA+1.0))*TIP
      VISCR=VISREF*((TCR/TREF)**1.5)*(TREF+TSJT)/(TCR+TSUT)
      TAU=RHOP*DIAP**2/18.0/VISCR
      RECR=DIAP*RHOCR*VCR/VISCR/2.0
      WRITE(6,5020) DELTA,TAU,RECR
      WRITE(6,4090)
      DO 110 I=1,MX
      DO 110 K=1,KMX
      AL(I,K)=AL(I,K)*CONV
      BETA(I,K)=BETA(I,K)*CONV
      V(I,K)=V(I,K)*VCR
  110 CONTINUE
C
C  DETERMINE AIR VELOCITIES AND PROPERTIES AT PARTICLE LOCATION.
C  SET UP TO START TRACE AND INITIALIZE.
C
      CALL BLOCAT(R,Z,MX,KMX,YR,IP,KP,NOUT)
      IF(NOUT.EQ.0) GO TO 130
      WRITE(6,4040) NOUT
      GO TO 105
  130 CALL POLATE(R,Z,V,YR(1),YR(5),IP,KP,VP,DD)
      CALL POLATE(R,Z,BETA,YR(1),YR(5),IP,KP,BETAP,DD)
      CALL POLATE(R,Z,AL,YR(1),YR(5),IP,KP,ALPP,DD)
      VR(1)=VP*DCOS(BETAP)*DSIN(ALPP)
      VJ(1)=VP*DSIN(BETAP)
      VZ(1)=VP*DCOS(BETAP)*DCOS(ALPP)
      TEMPA(1)=TIP*(1.-VP**2/CP/2./TIP)
      RHOA(1)=RHOIP*(TEMPA(1)/TIP)**(1./GAMMA-1.)
      VISTAR(1)=VISREF*(TEMPA(1)/TREF)**1.5*(TREF+TSUT)/(TEMPA(1)+TSUT)
```

```
C
C  INITIALIZE FOR FIRST STEP.
C
      RPART=DIAP/2.
      N=6
      M=1
      NTIME=-1
      T=0.0
      TS=T
      DO 610 I=1,4
      VR(I)=VR(1)
      VU(I)=VU(1)
      VZ(I)=VZ(1)
      RHOA(I)=RHOA(1)
      TEMPA(I)=TEMPA(1)
  610 VISTAR(I)=VISTAR(1)
      DO 620 I=1,N
  620 YRS(I)=YR(I)
C
C  INTEGRATE USING RUNGE-KUTTA METHOD.
C
  625 VDIFF=DSQRT((VR(2)-YR(2))**2+(VU(2)-YR(1)*YR(4))**2+(VZ(2)-YR(6))
     1 **2)
      RENOLD=RHOA(2)*VDIFF*DIAP/VISTAR(2)
      CALL RNUMBR(RENOLD,DGFC,CD)
      BCON=RHOA(2)*CD/RHOP/RPART/2.33333
  630 IF(RUNGE(N,YR,FR,T,H).NE.1) GO TO 640
      FR(1)=YR(2)
      FR(2)=YR(1)*YR(4)**2+BCON*VDIFF*(VR(2)-YR(2))
      FR(3)=YR(4)
      FR(4)=-2.0*YR(2)*YR(4)/YR(1)+BCON*VDIFF*(VU(2)-YR(1)*YR(4))/YR(1)
      FR(5)=YR(6)
      FR(6)=BCON*VDIFF*(VZ(2)-YR(6))
      GO TO 630
  640 CONTINUE
C
C  DETERMINE IF WALL INTERACTION OCCURRED.
C
      CALL BLOCAT(R,Z,MX,KMX,YR,IP,KP,NOUT)
      IF(NOUT.EQ.0) GO TO 700
      IF((NOUT.EQ.1).OR.(NOUT.EQ.3)) GO TO 780
      DO 645 I=1,6
  645 YR(I)=YRS(I)
      T=TS
      IF(NOUT.NE.2) GO TO 650
      RA(1)=R(IP,1)
      RA(2)=R(IP-1,1)
      ZA(1)=Z(IP,1)
      ZA(2)=Z(IP-1,1)
      WRITE(6,5000) NOUT
      CALL RBCH(YR,RA,ZA,T,H,ETA,NOUT)
      GO TO 750
  650 CONTINUE
```

```
      RA(1)=R(IP,KMX)
      RA(2)=R(IP-1,KMX)
      ZA(1)=Z(IP,KMX)
      ZA(2)=Z(IP-1,KMX)
      WRITE(6,5000) NOUT
      CALL RRCH(YR,RA,ZA,T,H,PTA,NOUT)
      GO TO 750

C
C  DETERMINE AIR VALUES AT NEW LOCATION.
C
  700 CALL POLATE(R,Z,   V,YR(1),YP(5),IP,KP,VP   ,DD)
      CALL POLATE(R,Z,BETA,YR(1),YP(5),IP,KP,BETAP,DD)
      CALL POLATE(R,Z,  AL,YR(1),YP(5),IP,KP, ALPP,DD)
      VR(4)=VP*DCOS(BETAP)*DSIN(ALPP)
      VU(4)=VP*DSIN(BETAP)
      VZ(4)=VP*DCOS(BETAP)*DCOS(ALPP)
      TEMPA(4)=1.0-(VP**2)/2.0/CP/TIP
      RHOA(4)=RHOIP*(TEMPA(4)/TIP)**(1.0/(GAMMA-1.0))
      VISTAR(4)=VISREF* (TEMPA(4)/TREF)**1.5*(TREF+TSUT)/(TEMPA(4)+TSUT)
C
C  TEST AIR VALUES USED, IF INCORRECT,RESET INTEGRATION VALUES
C  AND GO TO 750.
C
      IF((DABS(VR(4)-VR(3)).LT.1.0E-4).AND.(DABS(VU(4)-VU(3)).LT.1.0E-4)
     1 .AND.(DABS(VZ(4)-VZ(3)).LT.1.0E-4)) GO TO 750
      VR(2)=(VR(4)+VR(1))/2.0
      VU(2)=(VU(4)+VU(1))/2.0
      VZ(2)=(VZ(4)+VZ(2))/2.0
      VR(3)=VR(4)
      VU(3)=VU(4)
      VZ(3)=VZ(4)
      RHOA(2)=(RHOA(4)+RHOA(1))/2.0
      TEMPA(2)=(TEMPA(4)+TEMPA(2))/2.0
      VISTAR(2)= (VISTAR(4)+VISTAR(1))/2.0
      T=TS
      DO 710 I=1,N
  710 YR(I)=YRS(I)
      IF(M.GT.100) GO TO 800
      M=M+1
      GO TO 625

C
C  COMPLETE INTEGRATION STEP,  IF REQUIRED, WRITE OUTPUT.

  750 M=1
      L=L+1
      TS=T
      DO 760 I=1,N
  760 YRS(I)=YR(I)
      VR(2)=1.5*VR(4)-0.5*VR(1)
      VR(3)=2.0*VR(4)-VR(1)
      VR(1)=VR(4)
      VR(4)=VR(3)
      VU(2)=1.5*VU(4)-0.5*VU(1)
```

```
      VU(3)=2.0*VU(4)-VU(1)
      VU(1)=VU(4)
      VU(4)=VU(3)
      VZ(2)=1.5*VZ(4)-0.5*VZ(1)
      VZ(3)=2.0*VZ(4)-VZ(4)
      VZ(1)=VZ(4)
      VZ(4)=VZ(3)
      RHOA(2)=1.5*RHOA(4)-0.5*RHOA(1)
      RHOA(3)=2.0*RHOA(4)-RHOA(1)
      RHOA(1)=RHOA(4)
      RHOA(4)=RHOA(3)
      TEMPA(2)=1.5*TEMPA(4)-0.5*TEMPA(1)
      TEMPA(3)=2.0*TEMPA(4)-TEMPA(1)
      TEMPA(1)=TEMPA(4)
      TEMPA(4)=TEMPA(3)
      VISTAR(2)=1.5*VISTAR(4)-0.5*VISTAR(1)
      VISTAR(3)=2.0*VISTAR(4)-VISTAR(1)
      VISTAR(1)=VISTAR(4)
      VISTAR(4)=VISTAR(3)
      NTIME=NTIME-1
  780 IF((NOUT.EQ.1).OR.(NOUT.EQ.3).OR.(T.GT.TMAX)) GO TO 800
      IF(NTIME.GT.0) GO TO 625
      WRITE(6,4050) T,(YR(I),I=1,6),RENOLD
      NTIME=NSTEP-1
      GO TO 625
  800 CONTINUE
      WRITE(6,4060) T,(YR(I),I=1,6),RENOLD
      WRITE(6,4070) M
  805 CONTINUE
      GO TO 105
C
C  FORMAT STATEMENTS
C
 1000 FORMAT(2I5,6F10.5)
 1005 FORMAT(8F10.5)
 1010 FORMAT(8F10.5)
 1015 FORMAT(8E10.4)
 2000 FORMAT(1H ,8G16.7)
 2010 FORMAT(9H R ARRAY)
 2020 FORMAT(9HORC ARRAY)
 2030 FORMAT(12HOALPHA ARRAY)
 2040 FORMAT(8HOZ ARRAY)
 2050 FORMAT(7HOLAMBDA)
 2060 FORMAT(1H1,25H THE ORTHOGONAL NUMBER IS,I3///)
 2070 FORMAT(46HOA SOLUTION CANNOT BE OBTAINED AT THIS STATION)
 2080 FORMAT(1HO,43H AFTER 10, V(I,1) .GT. 2. CP*CIP.  E(I,1) =,F15.2)
 2081 FORMAT(1HO,49H DO 20 LOOP, V(I,K+1) .GT. 2.*CP*TIP.  V(I,K+1) =,
     1 F15.2)
 2090 FORMAT(11HOBETA ARRAY/(1X,8G16.7))
 2100 FORMAT(8HOV ARRAY/(1X,8G16.7))
 2110 FORMAT(42HOTHE PASSAGE IS CHOKED WITH A MASS FLOW OF,G16.7)
 2120 FORMAT(7HOWTFLES/(1X,8G16.7))
 2130 FORMAT(1H1,11H INPUT DATA,/,8X,2HMX,7X,3HKMX, 8X,2HZ1,13X,2HZ2,
```

```
     1  13X,2HR1,13X,2HR2,/,(2I10,4F15.5))
2140 FORMAT(1H0,10X,5HGAMMA,13X,2HCP,12X,3HTTP,10X,5HRHOTP,11X,
     1  4HWTFL,/,(F15.4,2F15.3,2E15.5))
2150 FORMAT(1H0,5X,14HLAMBDA, K = 1,,I3,/,(8G15.5))
2160 FORMAT(1H0,5X,13HALPHA, I = 1,,I3,/,(8G15.5))
2170 FORMAT(1H0,5X,13HRC, K = 1,,I3,/,(8G15.5))
3000 FORMAT(18A4)
3010 FORMAT(E20.5,4F10.3)
3020 FORMAT(F10.2,3F10.4,2F10.3)
3030 FORMAT(6F10.3)
3080 FORMAT(I5)
4000 FORMAT(1H1,18A4)
4010 FORMAT(1H0,13X,5HVISREF,11X,4HTREF,11X,4HTSUT,11X,4HRAIR,11X,
     1  4HDGFC,/,(E20.5,4F15.3))
4020 FORMAT(1H0,10X,4HRHOP,11X,4HDIAP,14X,1HH,11X,4HTMAX,10X,5HETA-N,
     1  10X,5HETA-T,/,(F15.2,3E15.4,2F15.3))
4030 FORMAT(1H0, 9X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,5HYR(4),10X,
     1  5HYR(5),10X,5HYR(6),/(6F15.3))
4040 FORMAT(1H0,62H PARTICLE NOT IN PASSAGE AT FIRST POINT GIVEN, GO TO
     1  NEXT CASE)
4050 FORMAT(1H ,E15.2,7F15.4)
4060 FORMAT(1H0,E15.2,7F15.4)
4070 FORMAT(1H0,4H M =,I10)
4080 FORMAT(17HOPRINT DATA EVERY,I7,2X,7HSTEP(S))
4090 FORMAT(1H0,13X,1HT,10X,5HYR(1),10X,5HYR(2),10X,5HYR(3),10X,
     1  5HYR(4),10X,5HYR(5),10X,5HYR(6),9X,6HRENOLD)
5000 FORMAT(29H PARTICLE BOUNCED OFF SURFACE,I5)
5020 FORMAT(32H0SIMILARITY PARAMETERS.  DELTA =,F10.4,5X,5HTAU =,E12.4,
     1  5X,6HRECR =,E12.4)
      END

        SUBROUTINE PARC(X,Y,A,B,C)
        IMPLICIT REAL*8 (A-H,O-Z)
C
C  SUBROUTINE PARC CALCULATES COEFFICIENTS A, B, C OF THE PARABOLA
C  Y = A*X**2 + B*X + C, PASSING THROUGH THREE GIVEN X,Y POINTS
C
        DIMENSION X(3),Y(3)
        C1=X(2)-X(1)
        C2=(Y(2)-Y(1))/(X(2)-X(1))
        A=(C1*C2-Y(3)+Y(1))/C1/(X(2)-X(3))
        B=C2-(X(1)+X(2))*A
        C=Y(1)-X(1)*B-X(1)**2*A
        RETURN
        END
```

The function routine RUNGE has been removed from the published form of this report to protect the copyright of the authors of Reference 5.

```
      SUBROUTINE CONTIN(XEST,YCALC,IND,JZ,YGIV,XDEL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(3),Y(3)
      NCALL=NCALL+1
      IF(IND.NE.1.AND.NCALL.GT.50) GO TO 160
      GO TO (10,30,40,50,60,80,130),IND
C*****FIRST CALL
   10 NCALL=1
      IF(YCALC.GT.YGIV.AND.JZ.EQ.1) GO TO 20
      IND=2
      Y(1)=YCALC
      X(1)=XEST
      XEST=XEST+XDEL
      RETURN
   20 IND=3
      Y(3)=YCALC
      X(3)=XEST
      XEST=XEST-XDEL
      RETURN
C*****SECOND CALL
   30 IND=4
      Y(2)=YCALC
      X(2)=XEST
      XEST=XEST+XDEL
      RETURN
   40 IND=5
      Y(2)=YCALC
      X(2)=XEST
      XEST=XEST-XDEL
      RETURN
C*****THIRD OR LATER CALL - FIND SUBSONIC OR SUPERSONIC SOLUTION
   50 Y(3)=YCALC
      X(3)=XEST
      GO TO 70
   60 Y(1)=YCALC
      X(1)=XEST
   70 IF(YGIV.LT.DMIN1(Y(1),Y(2),Y(3))) GO TO (90,95),JZ
   75 IND=6
      CALL PARC(X,Y,APA,BPB,CPC)
      DISCR=BPB**2-4.*APA*(CPC-YGIV)
      IF(DISCR.LT.0.0) GO TO 110
      IF(DABS(400.*APA*(CPC-YGIV)).LE.BPB**2) GO TO 78
      XEST=-BPB-DSIGN(DSQRT(DISCR),APA)
      IF(JZ.EQ.2.AND.APA.LT.0.0) XEST=-BPB-DSQRT(DISCR)
      XEST=XEST/2./APA
      GO TO 79
   78 ACB2=APA/BPB*(CPC-YGIV)/BPB
      IF(DABS(ACB2).LE.1.E-8) ACB2=0.0
      XEST=-(CPC-YGIV)/BPB*(1.+ACB2+2.*ACB2**2)
   79 IF(XEST.GT.X(3)) GO TO 95
      IF(XEST.LT.X(1)) GO TO 90
      RETURN
C*****FOURTH OR LATER CALL - (NOT CHOKED)
```

```
   80 IF(XEST.GT.X(3)) GO TO 50
      IF(XEST.LT.X(1)) GO TO 60
      Y(2)=YCALC
      X(2)=XEST
      GO TO 70
C*****THIRD OR LATER CALL / SOLUTION EXISTS,
C********BUT RIGHT OR LEFT SHIFT REQUIRED.
   90 IND=5
C*****LEFT SHIFT
      Y(3)=Y(2)
      X(3)=X(2)
      Y(2)=Y(1)
      X(2)=X(1)
      XEST=X(1)-XDEL
      RETURN
   95 IND=4
C*****RIGHT SHIFT
      Y(1)=Y(2)
      X(1)=X(2)
      Y(2)=Y(3)
      X(2)=X(3)
      XEST=X(3)+XDEL
      RETURN
C*****THIRD OR LATER CALL - APPEARS TO BE CHOKED
  110 XEST=-BPB/2./APA
      IND=7
      IF(X(1).LE.XEST.AND.XEST.LE.X(3)) RETURN
      IF(XEST.LT.X(1)) GO TO 90
      GO TO 95
C*****FOURTH OR LATER CALL - PROBABLY CHOKED
  130 IF(YCALC.GE.YGIV) GO TO 80
      IND=10
      RETURN
C******NO SOLUTION FOUND IN 50 ITERATIONS
  160 IND=11
      RETURN
      END

      SUBROUTINE ENTGRL(DX,F,N,ANS)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION F(50),ANS(50)
      DO 10 I=1,N
      IF(I.EQ.1) ANS(1)=0.0
      IF(I.EQ.2) ANS(2)=DX*(F(1)+F(2))/2.0
      IF(I.EQ.3) ANS(3)=DX*(F(1)+4.0*F(2)+F(3))/3.0
      IF(I.EQ.4) ANS(4)=3.0*DX*(F(1)+3.0*F(2)+3.0*F(3)+F(4))/8.0
      IF(I.GT.4) ANS(I)=ANS(I-2)+DX*(F(I-2)+4.0*F(I-1)+F(I))/3.0
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BLOCAT(R,Z,MX,KMX,YR,IP,KP,NOUT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION R(21,21),Z(21,21),YR(6)
      NOUT=0
      DO 20 I=1,MX
      IF(DABS(Z(I,1)-Z(I,KMX)).LT.1.0E-12) GO TO 10
      A=(R(I,1)-R(I,KMX))/(Z(I,1)-Z(I,KMX))
      B=R(I,1)-A*Z(I,1)
      RTEST=A*YR(5)+B
      IF(RTEST.LE.YR(1)) GO TO 30
      GO TO 20
   10 IF(Z(I,1).GE.YR(5)) GO TO 30
   20 CONTINUE
   30 IP=I
      IF(IP.NE.1) GO TO 40
      NOUT=1
      RETURN
   40 IF(IP.NE.MX) GO TO 50
      IF(R(MX,1).LT.YR(1)) GO TO 50
      NOUT=3
      RETURN
   50 CONTINUE
      DO 70 K=1,KMX
      IF(DABS(Z(IP,K)-Z(IP-1,K)).LT.1.0E-12) GO TO 60
      A=(R(IP-1,K)-R(IP,K))/(Z(IP-1,K)-Z(IP,K))
      B=R(IP,K)-A*Z(IP,K)
      RTEST=A*YR(5)+B
      IF(YR(1).LE.RTEST) GO TO 80
      GO TO 70
   60 IF(Z(IP,K).GE.YR(5)) GO TO 80
   70 CONTINUE
   80 KP=K
      IF(KP.NE.1) GO TO 90
      NOUT=2
      RETURN
   90 IF(KP.NE.KMX) GO TO 100
      IF(DABS(Z(IP,KP)-Z(IP-1,KP)).LT.1.0E-12) GO TO 110
      IF(YR(1).LT.RTEST) GO TO 100
      NOUT=4
  100 RETURN
  110 IF(YR(5).GE.Z(IP,KP)) NOUT=4
      RETURN
      END
```

```
      SUBROUTINE RBCH(YR,R,Z,T,H,ETA,NOUT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YR(6),R(2),Z(2),ETA(2)
      IF(DABS(Z(2)-Z(1)).LT.1.0E-12) GO TO 100
      IF(DABS(R(2)-R(1)).LT.1.0E-12) GO TO 200
      SM=(R(2)-R(1))/(Z(2)-Z(1))
      IF(DABS(YR(6)).LE.1.0E-12) GO TO 10
      IF(DABS(YR(2)).LE.1.0E-12) GO TO 20
      PM=YR(2)/YR(6)
      ZB=(Z(1)*SM-YR(5)*PM+YR(1)-R(1))/(SM-PM)
      RB=SM*(ZB-Z(1))+R(1)
      GO TO 30
   10 ZB=YR(5)
      RB=SM*(ZB-Z(1))+R(1)
      GO TO 30
   20 RB=YR(1)
      ZB=(RB-R(1))/SM+Z(1)
      GO TO 30
  100 IF(DABS(YR(2)).LT.1.0E-12) GO TO 110
      PM=YR(2)/YR(6)
      RB=PM*(Z(1)-YR(5))+YR(1)
      ZB=Z(1)
      GO TO 30
  110 RB=YR(1)
      ZB=Z(1)
      GO TO 30
  200 IF(DABS(YR(6)).LT.1.0E-12) GO TO 250
      PM=YR(2)/YR(6)
      RB=R(1)
      ZB=(RB-YR(1))/PM+YR(5)
      GO TO 30
  250 RB=R(1)
      ZB=YR(5)
   30 DBP=DSQRT((RB-YR(1))**2+(ZB-YR(5))**2)
      VM=DSQRT(YR(2)**2+YR(6)**2)
      DT=H-DBP/VM
      TB=YR(3)+YR(4)*DBP/VM
      GAMMA=DATAN2((R(2)-R(1)),(Z(2)-Z(1))) -1.5707963
      ALPHA=DATAN2(YR(2),YR(6))
      VN=VM*DCOS(GAMMA-ALPHA)
      VTR=VM*DSIN(GAMMA-ALPHA)
      VNP=-VN*ETA(1)
      VTRP=VTR*ETA(2)
      YR(4)=YR(4)*ETA(2)
      VMP=DSQRT(VNP**2+VTRP**2)
      BETA=DATAN2(VTRP,-VNP)
      YR(2)=-VMP*DSIN(GAMMA+BETA)
      YR(6)=-VMP*DCOS(GAMMA+BETA)
      YR(1)=RB+YR(2)*DT
      YR(3)=TB+YR(4)*DT
      YR(5)=ZB+YR(5)*DT
      T=T+H
      RETURN

      END
```

```
      SUBROUTINE SPLINT (X,Y,N,Z,MAX,YINT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(50),Y(50),S(50),A(50),B(50),C(50),F(50),W(50),SB(50),      0020
     1G(50),EM(50),Z(50),YINT(50)
      COMMON Q
      INTEGER Q
      DO 10 I=2,N                                                            0060
      S(I)=X(I)-X(I-1)
   10 CONTINUE
                                                                             0080
      NO=N-1                                                                 0090
      DO 20 I=2,NO                                                           0100
      A(I)=S(I)/6.0                                                          0110
      B(I)=(S(I)+S(I+1))/3.0                                                 0120
      C(I)=S(I+1)/6.0                                                        0130
   20 F(I)=(Y(I+1)-Y(I))/S(I+1)-(Y(I)-Y(I-1))/S(I)                           0140
      A(N)=-.5                                                               0150
      B(1)=1.0                                                               0160
      B(N)=1.0                                                               0170
      C(1)=-.5                                                               0180
      F(1)=0.0                                                               0190
      F(N)=0.0                                                               0200
      W(1)=B(1)                                                              0210
      SB(1)=C(1)/W(1)                                                        0220
      G(1)=0.0                                                               0230
      DO 30 I=2,N                                                            0240
      W(I)=B(I)-A(I)*SB(I-1)                                                 0250
      SB(I)=C(I)/W(I)                                                        0260
   30 G(I)=(F(I)-A(I)*G(I-1))/W(I)                                           0270
      EM(N)=G(N)                                                             0280
      DO 40 I=2,N                                                            0290
      K=N+1-I                                                                0300
   40 EM(K)=G(K)-SB(K)*EM(K+1)                                               0310
      DO 90 I=1,MAX                                                          0320
      K=2
      IF(Z(I)-X(1)) 60,50,70
   50 YINT(I)=Y(1)
      GO TO 90
   60 IF(Z(I).LT.(1.1*X(1)-.1*X(2)))WRITE (6,1000)Z(I)
      GO TO 85
 1000 FORMAT (17H OUT OF RANGE Z =F10.6)
   65 IF(Z(I).GT.(1.1*X(N)-.1*X(N-1))) WRITE (6,1000)Z(I)
      K=N
      GO TO 85
   70 IF(Z(I)-X(K)) 85,75,80
   75 YINT(I)=Y(K)
      GO TO 90
                                                                             0410
   80 K=K+1                                                                  0420
      IF(K-N) 70,70,65
   85 YINT(I) = EM(K-1)*(X(K)-Z(I))**3/6./S(K)+EM(K)*(Z(I)-X(K-1))**3/6.
     1/S(K)+(Y(K)/S(K)-EM(K)*S(K)/6.)*(Z(I)-X(K-1))+(Y(K-1)/S(K)-EM(K-1)
     2*S(K)/6.)*(X(K)-Z(I))
   90 CONTINUE                                                               0450
      MXA = MAXO(N,MAX)
```

Example

The example case presented here used the ft., slug, second system of units. The gas flow conditions correspond to inlet stagnation conditions of standard sea level air. The output that describes the mass flow through the turning vortex is contained on the first 5 pages of the output here. The output variables are the gas angle with respect to the meridional plane, β, and the velocity in terms of V/V_{cr}.

The particle used in the example has a specific gravity of 3 and a diameter of approximately 24 microns. Initially the particle has a velocity in the tangential direction with a velocity of 1000 rad/sec. The trajectory data indicates that this particle moves outward until it strikes the outer surface, where it bounces. The bounce drives the particle back into the inlet of the turning vortex.

The following pages contain a computer code sheet with the data arranged in the proper columns, and the output for this example.

VANPY Example Case for NASA Report

```
  10       7                0.0      0.00250      0.28500      0.28750  CARD GROUP A
     1.4       6012.4          518.7      0.00226      0.01560
-0.016666 -0.021167 -0.025833 -0.030250 -0.034750 -0.039250 -0.043333
       0.0       -4.800      -18.500      -30.000      -40.900      -50.260      -63.700      -75.000
   -86.900      -90.000
   214.60       214.77       214.94       215.11       215.28       215.45       215.60
EXAMPLE - TEST CASE
            0.106D-4          492.0          198.6       1716.48          1.0                    CARD
     187.2  0.776D-4          0.1D-4         0.1D-1           1.0          1.0                    GROUP
     0.305                    0.0            0.0          1000.0       0.0001          0.0          B
  10
Additional particle trajectories requires a Card Group B here.
These sets are stacked one after the other.                                                      CARD
                                                                                                 GROUP
                                                                                                   B
```

```
      IF(Q.EQ.16) WRITE(6,1010) N,MAX,(X(I),Y(I),Z(I),YINT(I),I=1,MXA)
 1010 FORMAT (2X21HNO. OF POINTS GIVEN #,I3,30H, NO. OF INTERPOLATED POI
     1NTS #,I3,/10X5HX     15X5HY     12X11HX-INTERPOL.9X11HY-INTERPOL./(4
     2E20.8))
  100 RETURN                                                            0460
      END                                                               0470

      SUBROUTINE POLATE(R,Z,A,RP,ZP,IP,KP,AP,DD)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION R(21,21),Z(21,21),A(21,21),D(2),DD(2)
      DO 10 I=1,2
      IA=IP+1-I
      IF(DABS(Z(IA,KP-1)-Z(IA,KP)).LT.1.0E-12) GO TO 5
      IF(DABS(R(IA,KP)-R(IA,KP-1)).LT.1.0E-12) GO TO 6
      AM=(R(IA,KP)-R(IA,KP-1))/(Z(IA,KP)-Z(IA,KP-1))
      B1=R(IA,KP-1)-AM*Z(IA,KP-1)
      B2=RP+ZP/AM
      ZA=(B2-B1)*AM/(AM**2+1.0)
      RA=B2-ZA/AM
      GO TO 10
    5 RA=RP
      ZA=Z(IA,KP-1)
      GO TO 10
    6 RA=R(IA,KP-1)
      ZA=ZP
   10 D(I)=DSQRT((RA-RP)**2+(ZA-ZP)**2)
      DT=D(1)+D(2)
      AA=(D(1)*A(IP,KP-1)+D(2)*A(IP-1,KP-1))/DT
      AB=(D(1)*A(IP,KP)+D(2)*A(IP-1,KP))/DT
      DO 20 K=1,2
      KA=KP-K+1
      RC=(D(1)*R(IP,KA)+D(2)*R(IP-1,KA))/DT
      ZC=(D(1)*Z(IP,KA)+D(2)*Z(IP-1,KA))/DT
   20 DD(K)=DSQRT((RP-RC)**2+(ZP-ZC)**2)
      DT=DD(1)+DD(2)
      AP=(DD(1)*AA+DD(2)*AB)/DT
      RETURN
      END

      SUBROUTINE RNUMBR(RENOLD,DGFC,CD)
      IMPLICIT REAL*8 (A-H,O-Z)
      IF(DABS(RENOLD).LT.1.0E-12) RENOLD=1.0E-12
      IF(RENOLD.LT.1.0) GO TO 26
      IF((RENOLD.GE.1.0).AND.(RENOLD.LT.1.0E3)) GO TO 27
      CD=DGFC*0.4
      RETURN
   26 CD=DGFC*(4.5+24.0/RENOLD)
      RETURN
   27 ARE=DLOG(RENOLD)
      CD=(28.5-24.0*ARE+9.0682*ARE**2-1.7713*ARE**3+0.1718*ARE**4
     1   -0.0065*ARE**5)*DGFC
      RETURN
      END
```

```
INPUT DATA
     NX        KPT        Z1                 Z2              A1             A2
     10          7        0.0                0.00250         0.28500        0.28750

        GAMMA          CP              TIP          RHCIP           WTFL
       1.4000       6012.400         518.700     0.226000-02     0.156000-01

    AC, K = 1, 7
 -0.166660-01   -0.211670-01   -0.258330-01   -0.302500-01   -0.347500-01   -0.392500-01   -0.433330-01

    ALPHA, I = 1, 10
 0.0            -4.8000         -18.560         -30.000         -40.500         -50.260         -63.700         -75.000
 -86.900        -90.000

    LAMBDA, K = 1, 7
  214.60         214.77          214.94          215.11          215.28          215.45          215.60
```

THE ORTHOGONAL NUMBER IS 1

P ARRAY						
[illegible]	0.3098670	0.3133330	0.3177500	0.3222500	0.3267500	0.3308330
Z ARRAY						
0.0	0.0	0.0	0.0	0.0	0.0	0.0
RC ARRAY						
-10000.00	-10000.00	-10000.00	-10000.00	-10000.00	-10000.00	-10000.00
SIGMA ARRAY						
0.0	0.0	0.0	0.0	0.0	0.0	0.0
LAMDA						
214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000
WEIGHTS						
0.15999940-01						
BETA ARRAY						
76.88497	76.89091	76.68843	76.72123	76.69327	76.66754	76.52442
V ARRAY						
0.7109197	0.7008106	0.6913817	0.6822178	0.6733004	0.6646227	0.6561600

THE ORTHOGONAL NUMBER IS 2

P ARRAY						
0.3041076	0.3085928	0.3132424	0.3176437	0.3221281	0.3266123	0.3306810
Z ARRAY						
0.38945730-02	0.42712070-02	0.46618470-02	0.50312520-02	0.54078020-02	0.57843520-02	0.61260080-02
RC ARRAY						
-0.16666000-01	-0.21167000-01	-0.25833000-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43333000-01
SIGMA ARRAY						
-4.800000	-4.800000	-4.800000	-4.800000	-4.800000	-4.800000	-4.800000
LAMDA						
214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000
WEIGHTS						
0.19999000-01						
BETA ARRAY						
67.91989	71.96740	74.59427	76.74401	78.36662	79.67638	81.16009
V ARRAY						
0.7434998	0.7182707	0.6950032	0.6723811	0.6502185	0.6286288	0.6071562

THE EIGENVALUE NUMBER IS 3

R ARRAY						
0.3033048	0.3075737	0.3119588	0.3161868	0.3204543	0.3247217	0.3285537
F ARRAY						
0.77881540-02	0.92163280-02	0.10696520-01	0.12090460-01	0.13524320-01	0.14954190-01	0.16246750-01
BC ARRAY						
-0.16666670-01	-0.21167000-01	-0.25833000-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43333000-01
ALPHA ARRAY						
-18.50000	-18.50000	-18.50000	-18.50000	-18.50000	-18.50000	-18.50000
LAMDA						
214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000
WTFIES						
0.19999990-01						
BETA ARRAY						
67.74031	71.81264	74.56205	76.72264	78.35637	79.67541	81.14913
V ARRAY						
0.7498319	0.7708777	0.7859763	0.6855867	0.6727389	0.6614598	0.6512845

THE EIGENVALUE NUMBER IS 4

R ARRAY						
0.3019992	0.3058812	0.3098720	0.3136973	0.3175644	0.3214915	0.3250275
F ARRAY						
0.10892660-01	0.13083490-01	0.15416490-01	0.17624980-01	0.19874980-01	0.22124980-01	0.24166480-01
BC ARRAY						
-0.16666000-01	-0.21167000-01	-0.25833000-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43333000-01
ALPHA ARRAY						
-30.00000	-30.00000	-30.00000	-30.00000	-30.00000	-30.00000	-30.00000
LAMDA						
214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000
WTFIES						
0.19999990-01						
BETA ARRAY						
67.68406	71.77150	74.50576	76.68524	78.33824	79.67337	81.12980
V ARRAY						
0.7519741	0.7719766	0.7859737	0.6911342	0.6788411	0.6681101	0.6584658

THE ORTHOGONAL NUMBER IS 5

S ARRAY

[illegible]	0.3034092	0.3070260	0.3103246	0.3137659	0.3171673	0.3202534

Y ARRAY

0.13411900-01	0.16358650-01	0.19413600-01	0.22305890-01	0.25252220-01	0.28198550-01	0.30871840-01

RC ARRAY

-0.19666000-01	-0.21167000-01	-0.25833000-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43333000-01

ALPHA ARRAY

-40.50000	-40.50000	-40.50000	-40.50000	-40.50000	-40.50000	-40.50000

LAMDA

214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6200

WEIGHTS

0.19999997E-01

BETA ARRAY

67.41747	71.58456	74.42156	76.63336	78.31270	79.66985	81.10263

V ARRAY

0.7598167	0.7319070	0.7127509	0.6987056	0.6871874	0.6772267	0.6683304

THE ORTHOGONAL NUMBER IS 6

S ARRAY

0.2981947	0.3010992	0.3040152	0.3068390	0.3097159	0.3125927	0.3152030

Y ARRAY

0.15315370-01	0.18776430-01	0.22364360-01	0.25760820-01	0.29221110-01	0.32681400-01	0.35821030-01

RC ARRAY

-0.19666000-01	-0.21167000-01	-0.25833000-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43333000-01

ALPHA ARRAY

-50.26000	-50.26000	-50.26000	-50.26000	-50.26000	-50.26000	-50.26000

LAMDA

214.6000	214.7700	214.9400	215.1100	215.2800	215.4500	215.6200

WEIGHTS

0.19999997E-01

BETA ARRAY

67.21461	71.44491	74.34251	76.57611	78.28482	79.66507	81.07321

V ARRAY

0.7656417	0.7381147	0.7181563	0.7069619	0.6982457	0.6871478	0.6798654

THE EIGENVALUE NUMBER IS 7

R ARRAY
0.2948862 0.2968789 0.2989456 0.3009879 0.3028968 0.3048906 0.3066597

Y ARRAY
0.17440890-01 0.21475520-01 0.25658620-01 0.29618700-01 0.33652880-01 0.37687070-01 0.41347420-01

RC ARRAY
-0.16666000-01 -0.21167000-01 -0.25833000-01 -0.30250000-01 -0.34750000-01 -0.39250000-01 -0.43333000-01

ALPHA ARRAY
-63.70000 -63.70000 -63.70000 -63.70000 -63.70000 -63.70000 -63.70000

T EMPDA
214.6000 214.7700 214.9400 215.1100 215.2800 215.4500 215.6000

WEIGHTS
0.15555550-01

BETA ARRAY
66.85662 71.19494 74.15082 76.47379 78.23152 79.65428 81.01676

W ARRAY
0.7742187 0.7465479 0.7229043 0.7211558 0.7120555 0.7045307 0.6980300

THE EIGENVALUE NUMBER IS 8

R ARRAY
0.2918199 0.2939785 0.2961861 0.2983293 0.3004940 0.3026587 0.3047155

Y ARRAY
0.19990110-01 0.23965740-01 0.27852750-01 0.31719240-01 0.35685610-01 0.40412570-01 0.44356450-01

RC ARRAY
-0.16666000-01 -0.21167000-01 -0.25833000-01 -0.30250000-01 -0.34750000-01 -0.39250000-01 -0.43333000-01

ALPHA ARRAY
-75.00000 -75.00000 -75.00000 -75.00000 -75.00000 -75.00000 -75.00000

T EMPDA
214.6000 214.7700 214.9400 215.1100 215.2800 215.4500 215.6000

WEIGHTS
0.19999990-01

BETA ARRAY
66.90581 70.94659 74.03871 76.37034 78.17692 79.64061 80.95943

W ARRAY
0.7844595 0.7606894 0.7453243 0.7350864 0.7275774 0.7216794 0.7168810

THE FOURTH ORDER IS 9

R ARRAY

0.2886319	0.2886447	0.2888591	0.2891351	0.2893793	0.2896227	0.2898435

Z ARRAY

0.19141610-01	0.23636020-01	0.28295200-01	0.32705730-01	0.37199150-01	0.41692560-01	0.45786590-01

RC ARRAY

-0.14666C00-01	-0.21167CC0-01	-0.25833C00-01	-0.30250000-01	-0.34750000-01	-0.39250000-01	-0.43233000-01

ALPHA ARRAY

-86.50000	-86.50000	-86.50000	-86.90000	-86.50000	-86.50000	-86.50000

LAMDA

214.6300	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000

WEIGHTS

0.15999999E-01

BETA ARRAY

66.10066	70.65931	73.65753	76.24603	78.10646	79.62079	80.68506

V ARRAY

0.7692561	0.7734400	0.7596654	0.7512289	0.7456455	0.7417505	0.7388870

THE FOURTH ORDER IS 10

R ARRAY

0.2850000	0.2850000	0.2850000	0.2850000	0.2850000	0.2850000	0.2850000

Z ARRAY

0.19141600-01	0.23667C00-01	0.28333C00-01	0.32750000-01	0.37250000-01	0.41750000-01	0.45833000-01

RC ARRAY

-10000.00	-10000.00	-10000.00	-10000.00	-10000.00	-10000.00	-10000.00

ALPHA ARRAY

-90.00000	-90.00000	-90.00000	-90.00000	-90.00000	-90.00000	-90.00000

LAMDA

214.6300	214.7700	214.9400	215.1100	215.2800	215.4500	215.6000

WEIGHTS

0.15999999E-01

BETA ARRAY

74.92972	75.69598	75.87861	76.05988	76.24356	76.42956	76.59583

V ARRAY

0.7627417	0.7627417	0.7627416	0.7627416	0.7627416	0.7627415	0.7627415

PROFILE - TEST CASE

RHOREF	TREF	TSLT	RAIR	EGFC
0.106800D-04	497.000	198.600	1716.480	1.000

RHOP	FIAP	P	TMAX	ETA-N	ETA-T
187.20	0.7760E-04	0.1000D-04	0.1000D-01	1.000	1.000

VR(1)	VR(2)	VR(3)	VR(4)	VR(5)	VR(6)
0.985	0.0	0.0	1000.000	0.001	0.0

PRINT DATA EVERY 10 STEPS(S)

STABILITY PARAMETERS. DELTA = 33.7580 TAU = 0.6554D-02 RECR = 0.59310 01

T	VR(1)	VR(2)	VR(3)	VR(4)	VR(5)	VR(6)	RENOLD
0.100D-04	0.9950	2.6586	0.010	598.1252	0.0010	1.3275	14.0939
0.100D-03	0.9949	30.1615	0.0995	588.4179	0.0011	1.3310	0.0062
0.100D-03	0.9104	84.6550	0.1075	582.6432	0.0012	1.3346	0.0061
0.200D-03	0.9167	81.8810	0.2726	526.0030	0.0014	1.3380	0.0060
0.300D-03	0.9251	109.4138	0.3538	478.5317	0.0015	1.3414	0.0059

PARTICLE POINTER OFF SURFACE 4

0.400D-03	0.9263	-110.3254	0.4213	467.0217	0.0014	-4.4910	0.0058
0.500D-03	0.9174	-87.2740	0.5116	516.3578	0.0010	-4.4876	0.0059
0.600D-03	0.9136	-62.4993	0.5959	556.6074	0.0006	-4.4841	0.0061
0.700D-03	0.9062	-36.2602	0.6834	584.6141	0.0002	-4.4806	0.0062
0.780D-03	0.9049	-21.3034	0.7329	593.5380	-0.0000	-4.4786	0.0063

P = 1

DISCUSSION OF RESULTS

The programs that are included here illustrate the general complexity of the problem of determining particle trajectories in a turbomachine. In tracing the particles all the way through a radial inflow turbine, the constantly changing nature of the flow requires that each region of the flow be considered individually. These programs are used to study the particle trajectories through each of these regions.

The programs have been developed over a three year period and some of the programs have subroutines that were stepping stones to the more complex routines that are explained in the section entitled, "General Numerical Techniques". Users of these programs might consider improvements by using the sophisticated subroutines instead, particularly the programs that do not allow variable particle restitution coefficients. The subroutine RESTCO, explained previously, can be used to describe in general the restitution coefficients. Experience has shown that the use of constant restitution coefficients less than 1.0 causes the particles to come to rest.

Several other programs, specifically the SCRL2D and STATOR programs could be improved with the addition of more realistic boundaries. In the scroll program, the solution of the gas flow is one-dimensional and the particle trajectories are two-dimensional. A better solution of the gas flow in this region might provide a slightly different particle trajectory pattern. In the stator program, the hub to shroud distance has been assumed constant, although in most real turbines, this distance is a function of the radius. Inclusion of this factor might lead to some interesting results in the study of particle trajectories in the radial turbine. These programs could be the basis of a very large program that could use a Monte Carlo Technique to study erosion rates from turbine internal surfaces.

CONCLUSIONS

This report has presented the computer programs that have been used to study the trajectories of particles in the radial inflow turbine. These programs can be used to investigate the trajectories of particles in radial inflow turbines, and provide information concerning the locations where particles strike the surfaces. This information can be used to predict the areas most subjected to erosion damage, in radial inflow turbines.

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Table 1. Coefficients of Polynomial, a_i, Describing Variation of Tangential Restitution Coefficient with Incidence Angle*

Coefficient of Number	Value of Coefficient
1	1.819024
2	5.117122
3	- 49.252858
4	131.035284
5	-174.424912
6	117.872348
7	- 24.478854
8	- 16.729740
9	11.230019
10	- 1.979996

*Angles expressed in radians

Table 2. Coefficients of Polynomial Describing Variation of Normal Restitution Coefficient with Incidence Angle*

Coefficient of Number	Value of Coefficient
1	5.721469
2	- 41.880784
3	178.168464
4	-424.388179
5	572.763055
6	-406.662526
7	87.142750
8	70.651124
9	- 50.498187
10	9.676744

*Angles expressed in radians

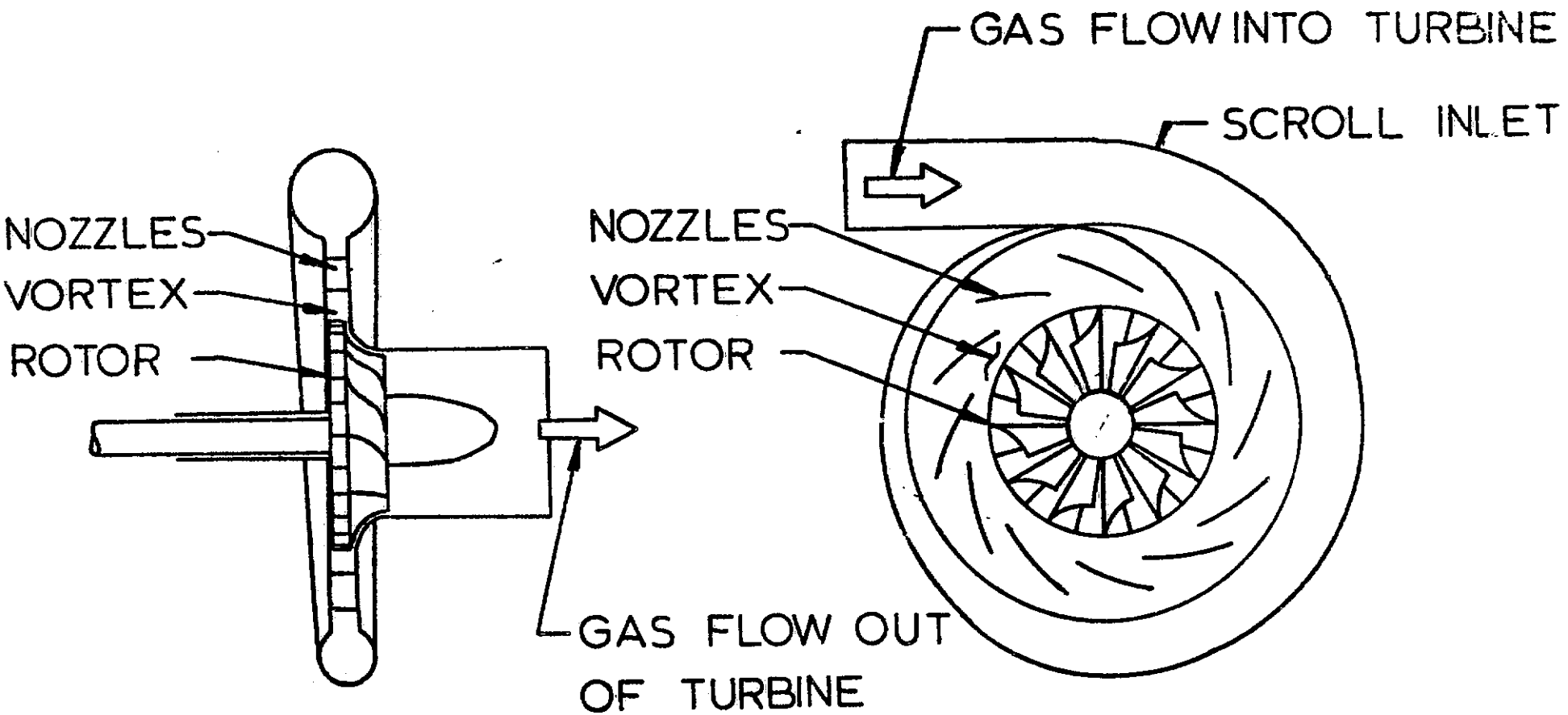


FIGURE 1. SCHEMATIC OF TYPICAL RADIAL INFLOW TURBINE

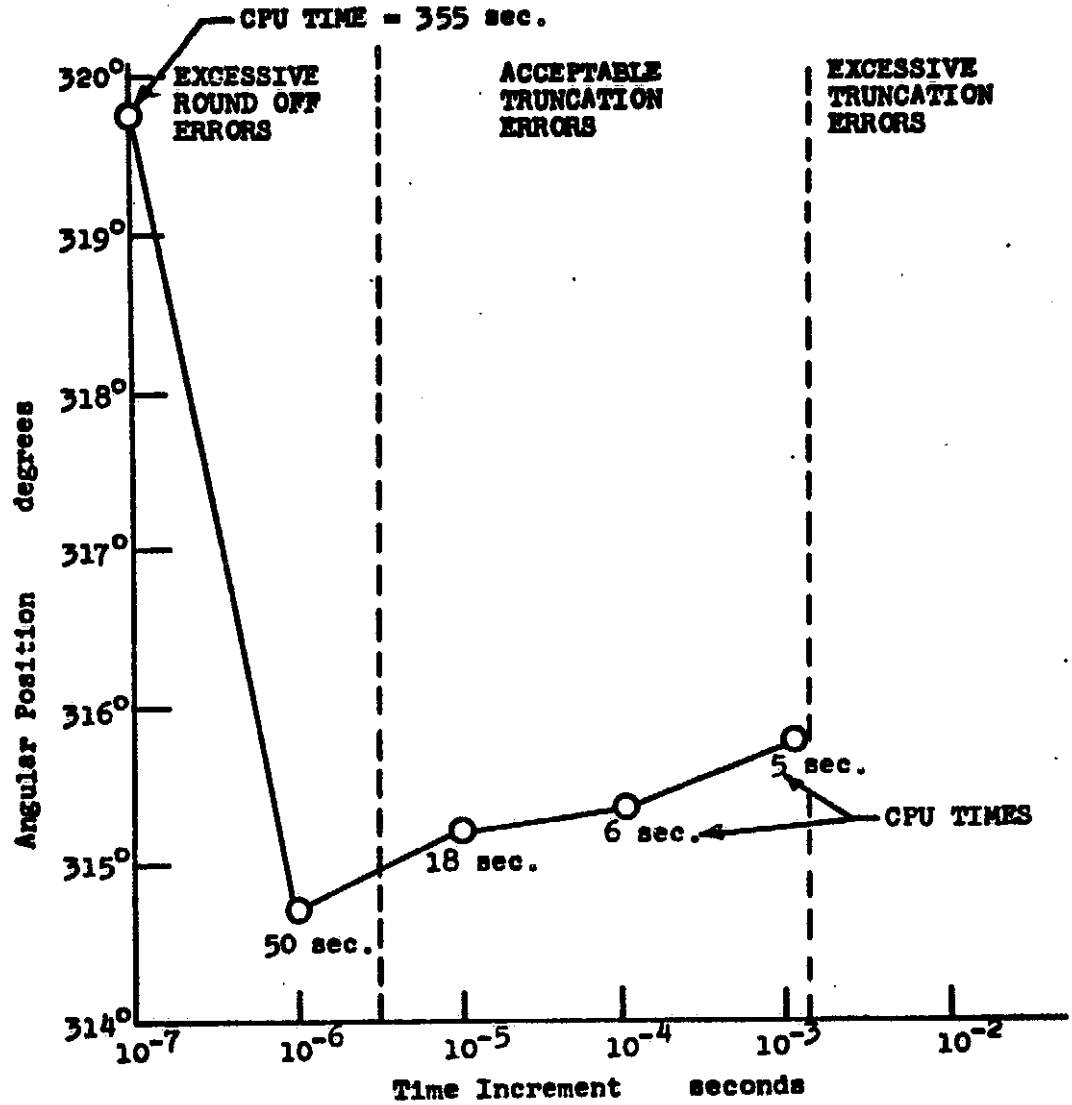


FIGURE 2. ROUND OFF AND TRUNCATION ERRORS INHERENT IN THE NUMERICAL PROCEDURE

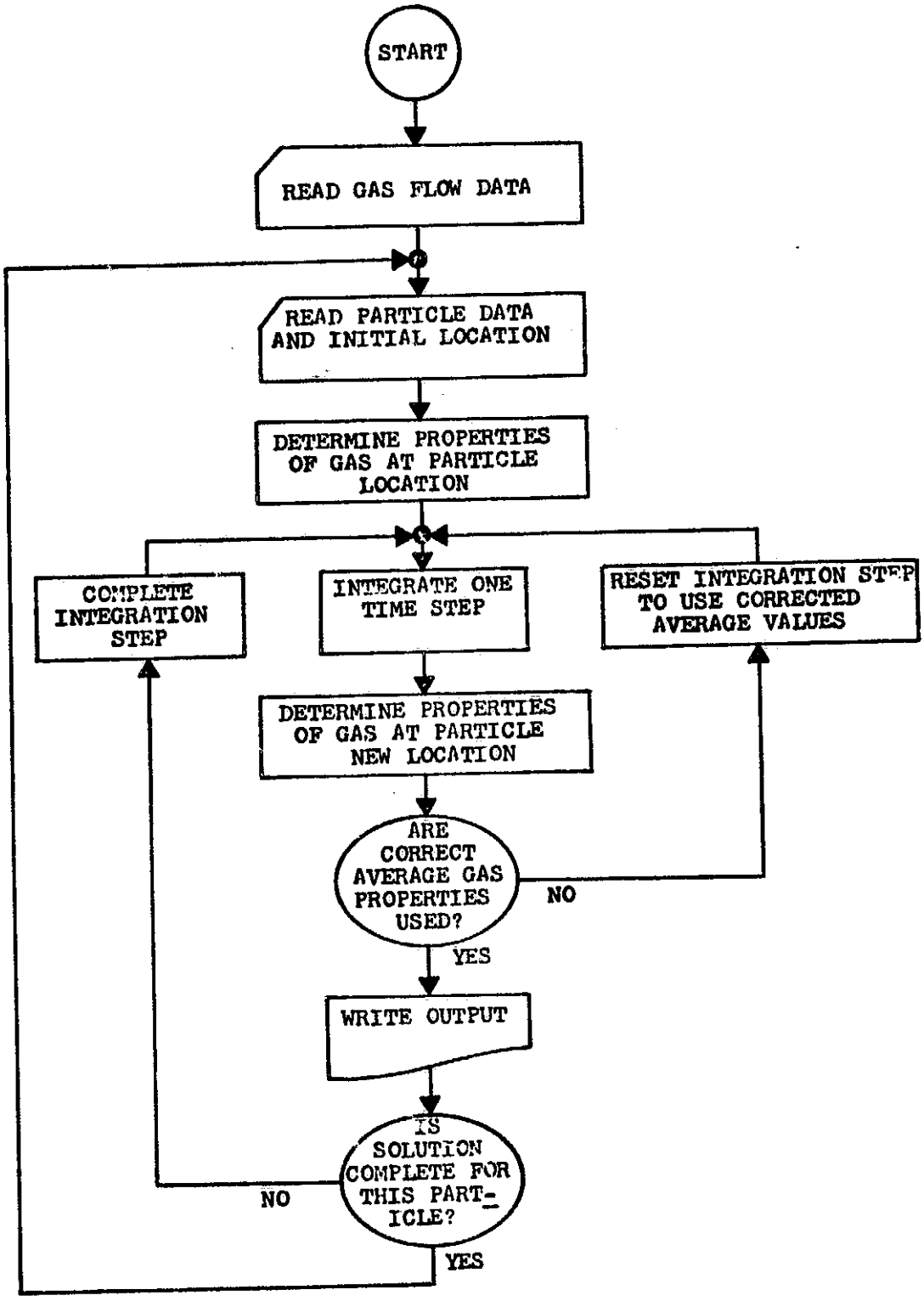


FIGURE 3. TYPICAL FLOW CHART

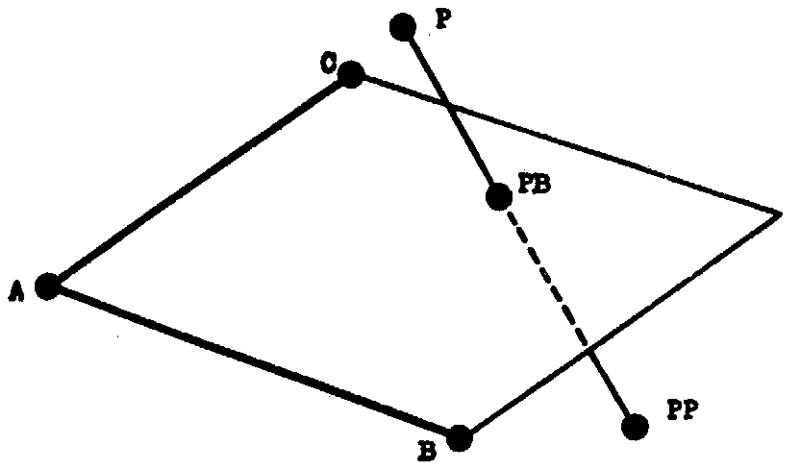


FIGURE 4. INTERSECTION OF TRAJECTORY WITH SURFACE

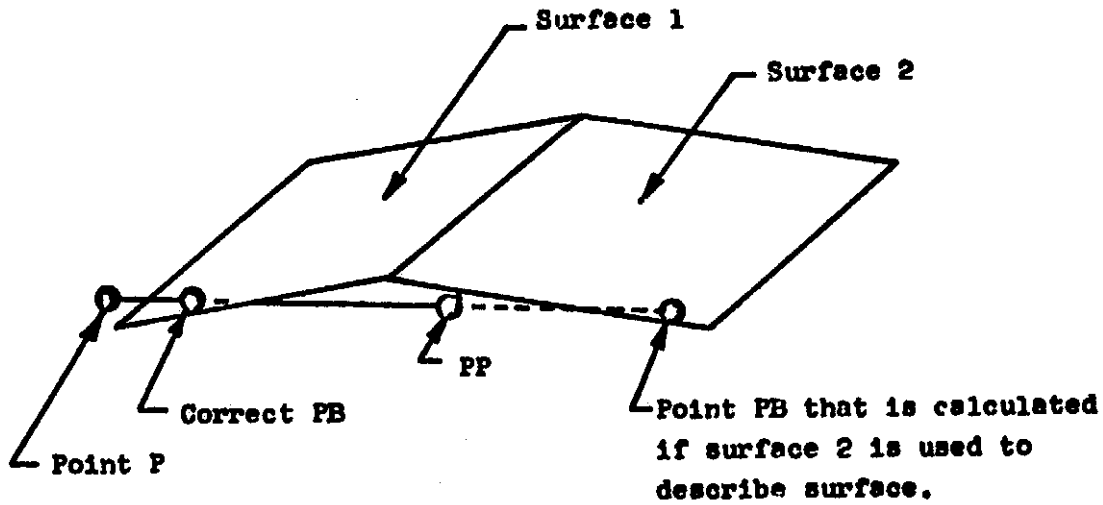


FIGURE 5. FIRST TYPE OF BOUNCE NEAR SURFACE NODES

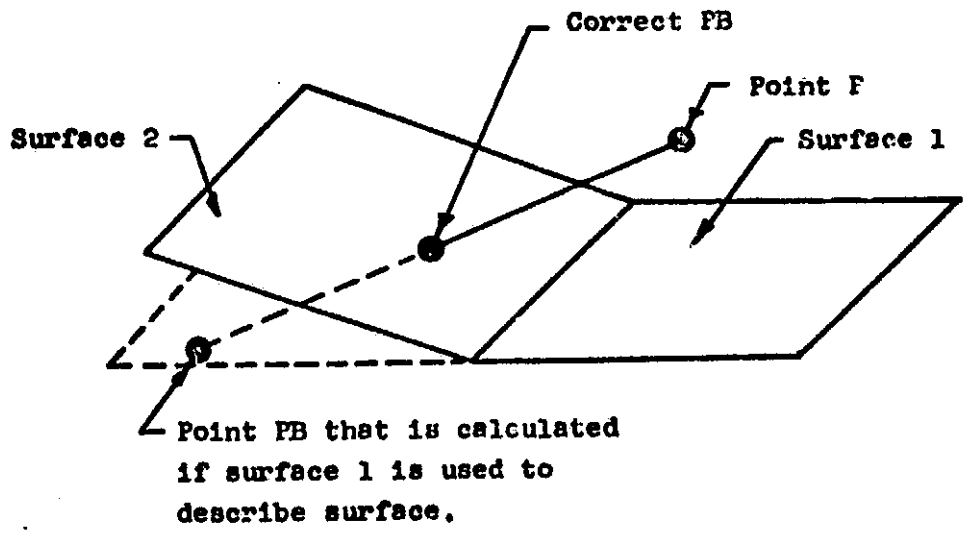


FIGURE 6. SECOND TYPE OF BOUNCE NEAR SURFACE NODES

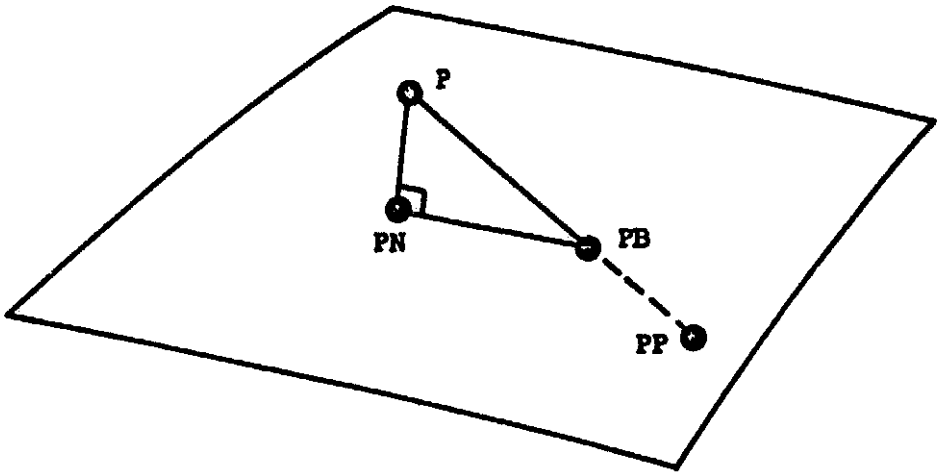


FIGURE 7. METHOD ONE POINTS

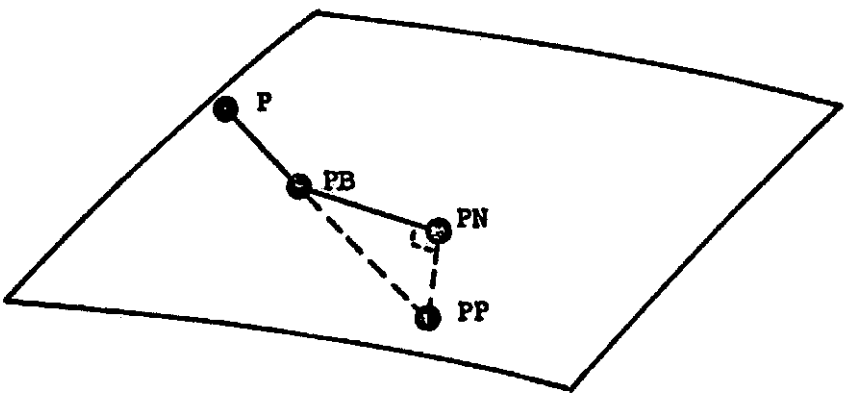


FIGURE 8. METHOD TWO POINTS

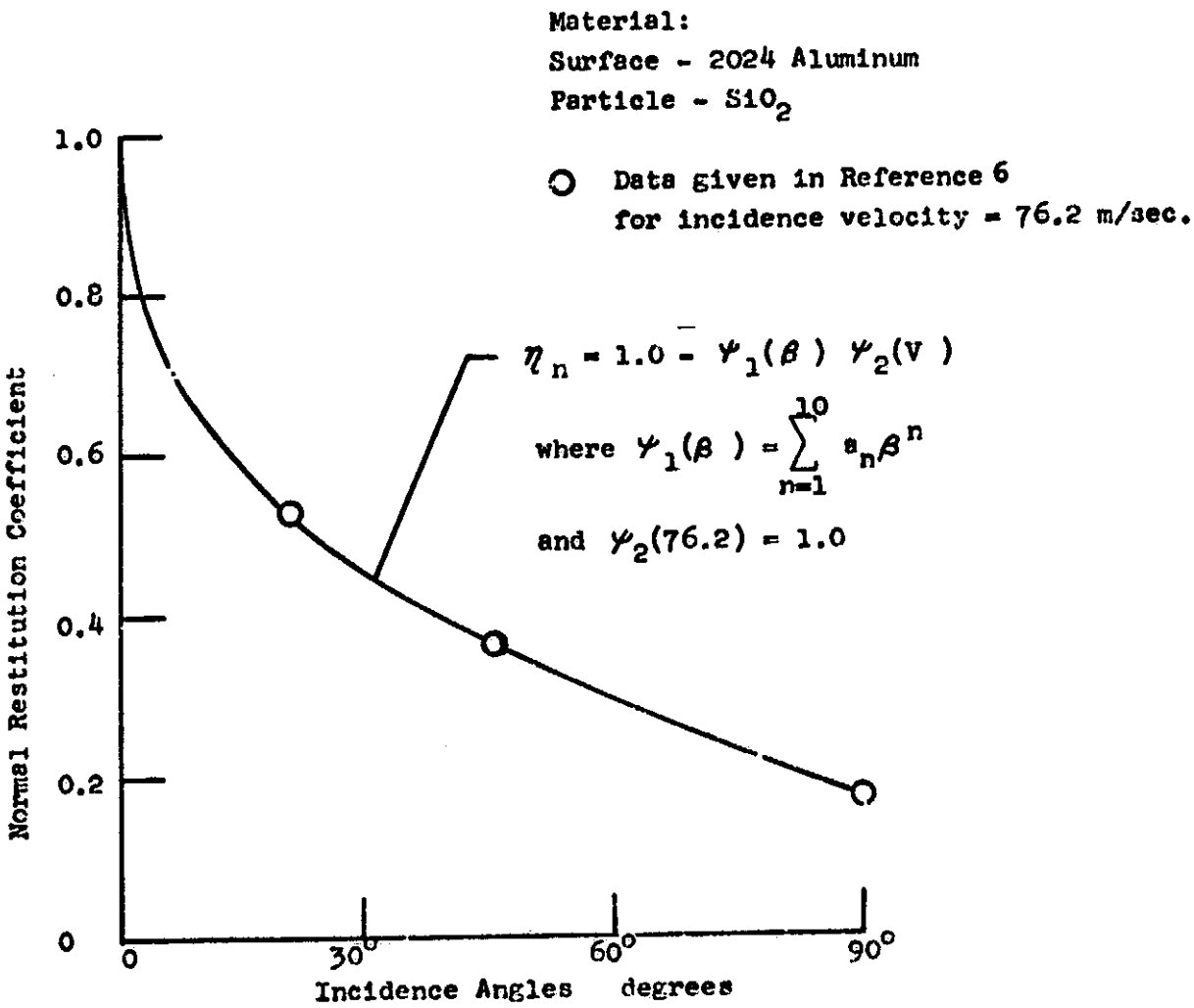


FIGURE 9. INFLUENCE OF INCIDENCE ANGLE ON NORMAL RESTITUTION COEFFICIENT

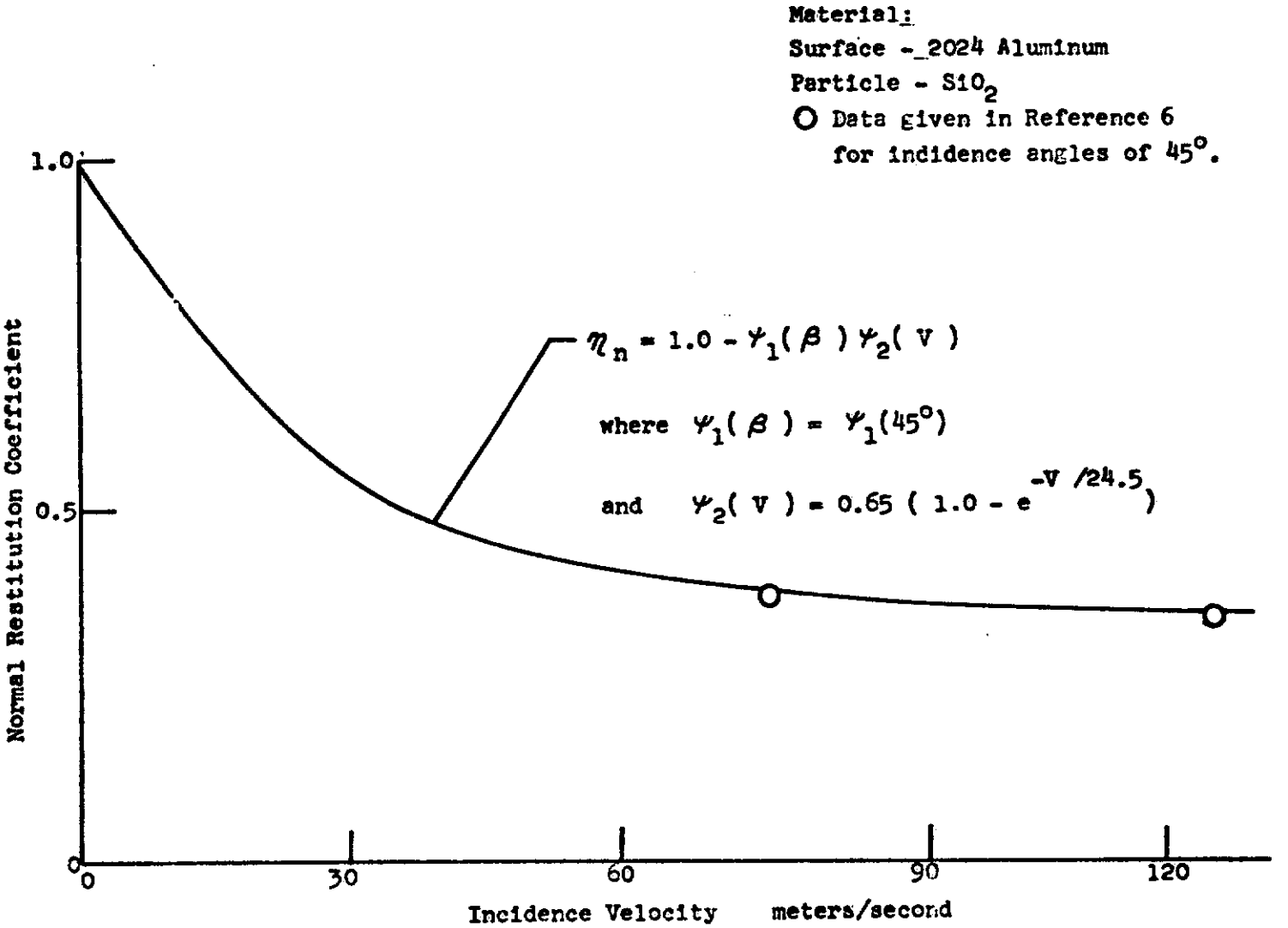


FIGURE 10. INFLUENCE OF INCIDENCE VELOCITY ON NORMAL RESTITUTION COEFFICIENT

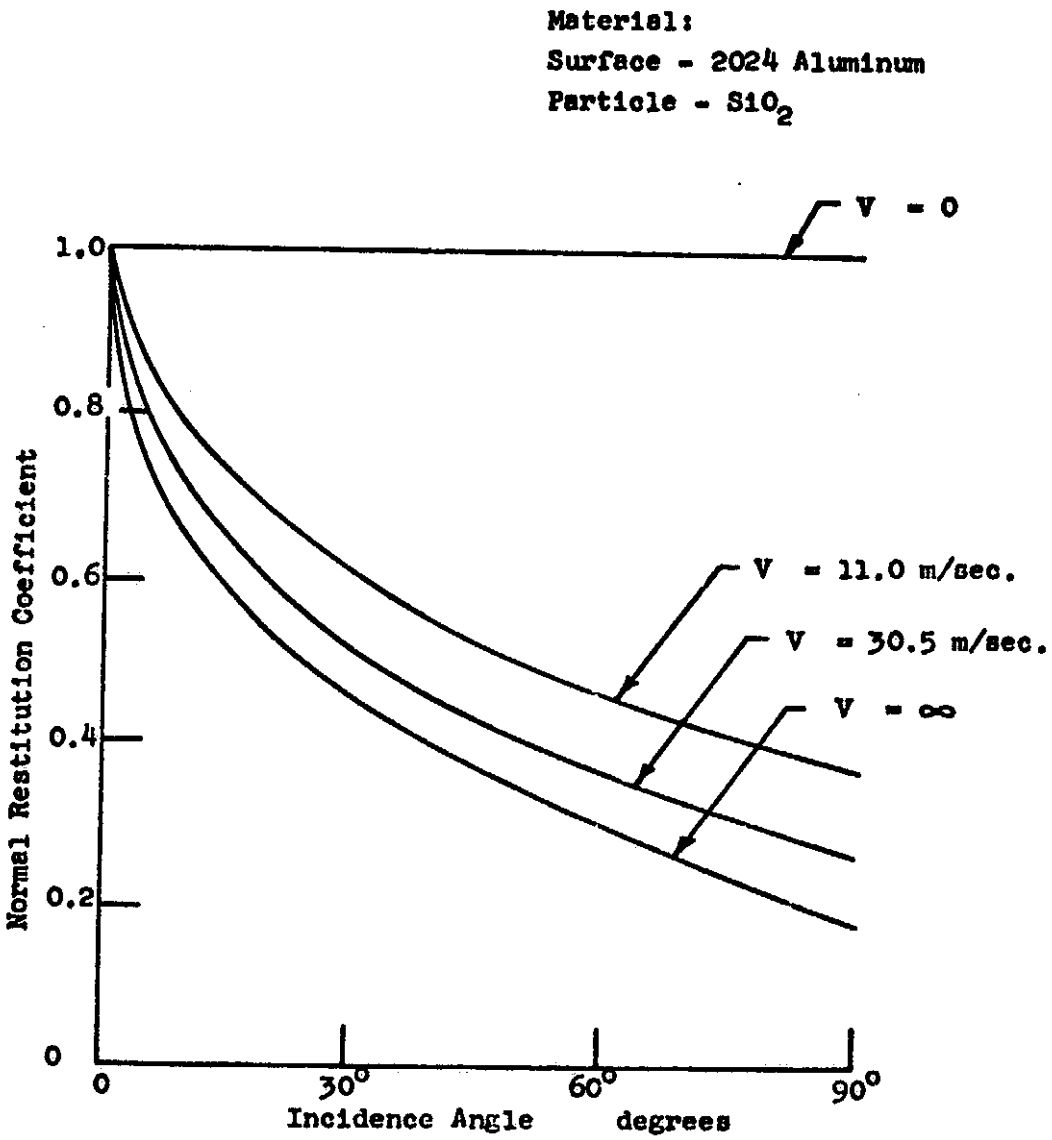


FIGURE 11. NORMAL RESTITUTION COEFFICIENT

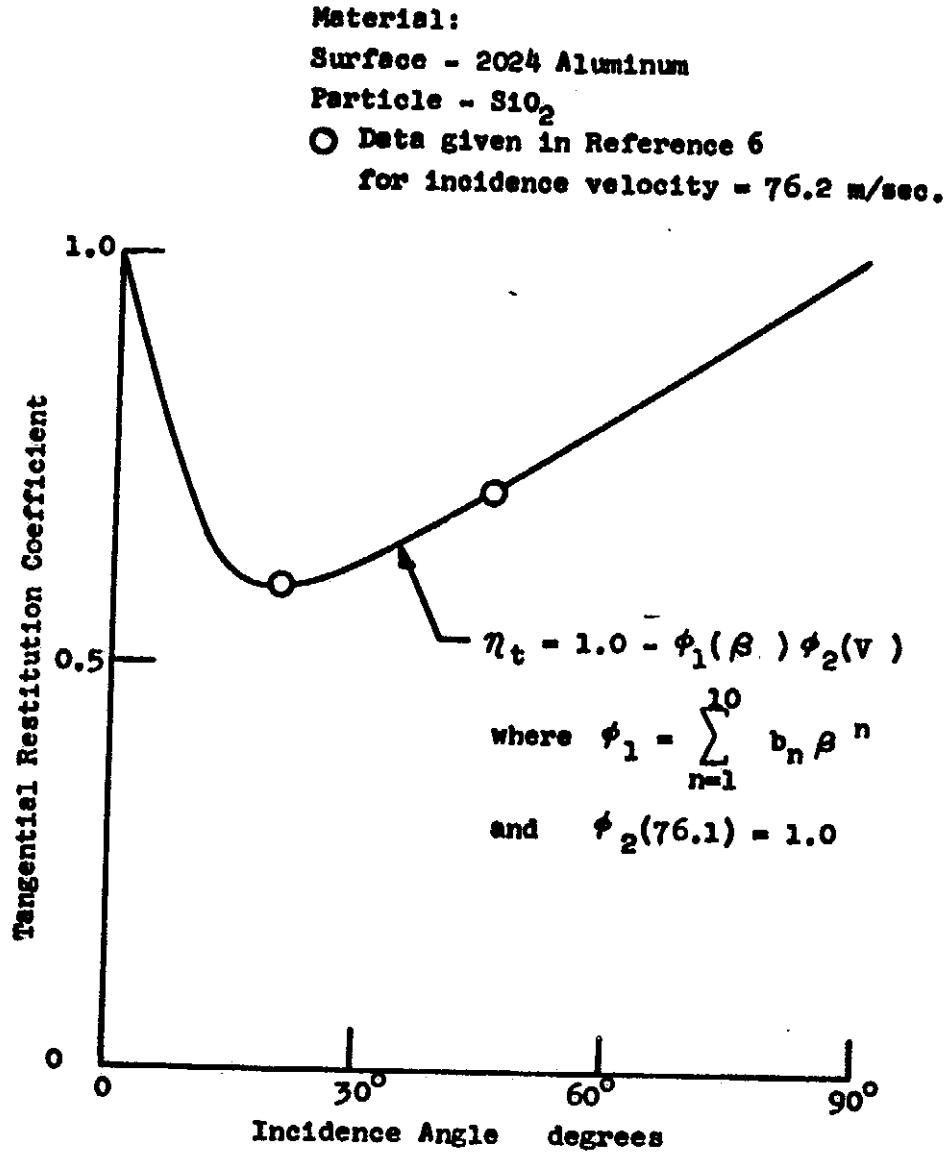


FIGURE 12. INFLUENCE OF INCIDENCE ANGLE ON TANGENTIAL RESTITUTION COEFFICIENT

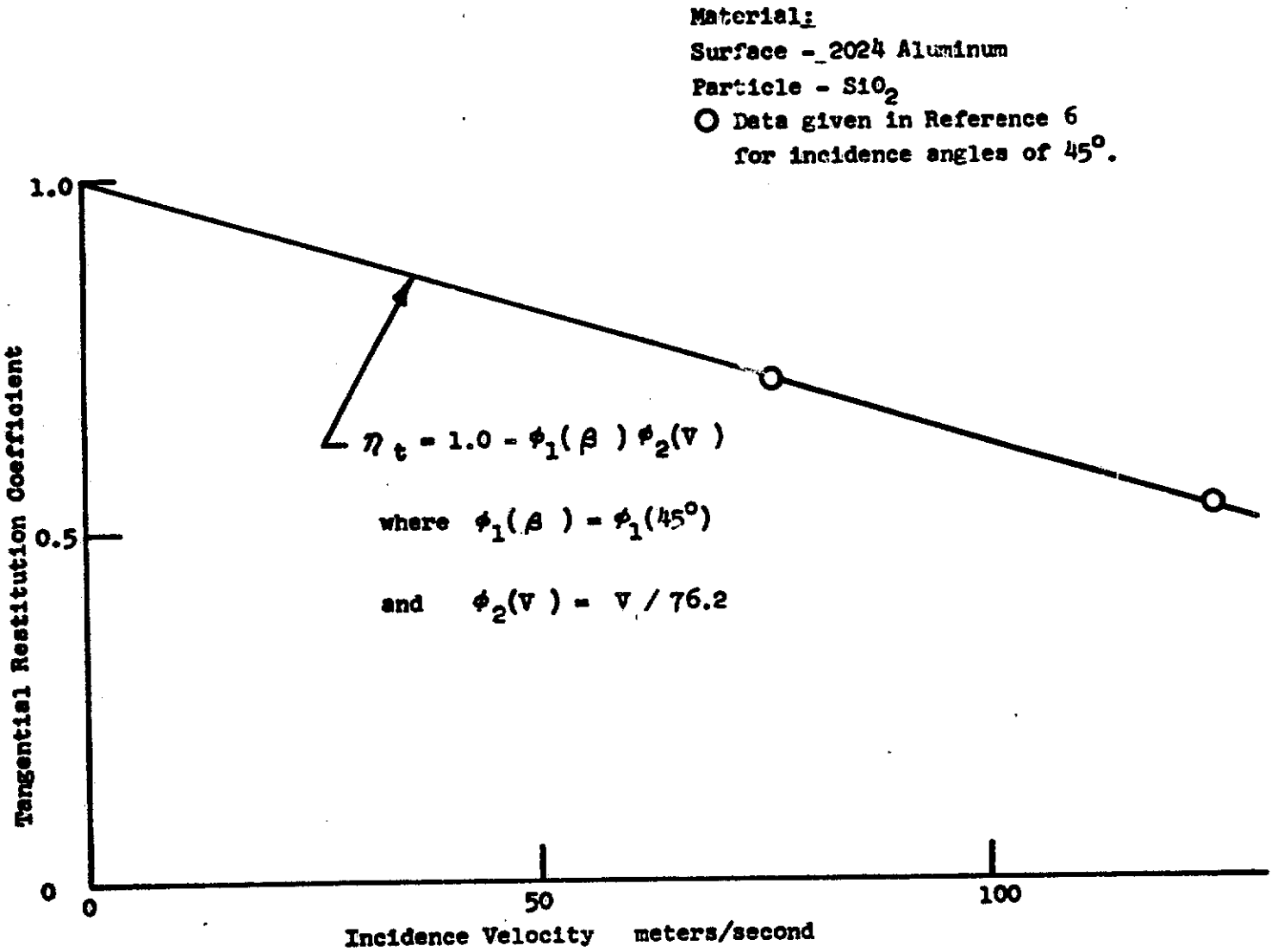


FIGURE 13. INFLUENCE OF INCIDENCE VELOCITY ON TANGENTIAL RESTITUTION COEFFICIENT

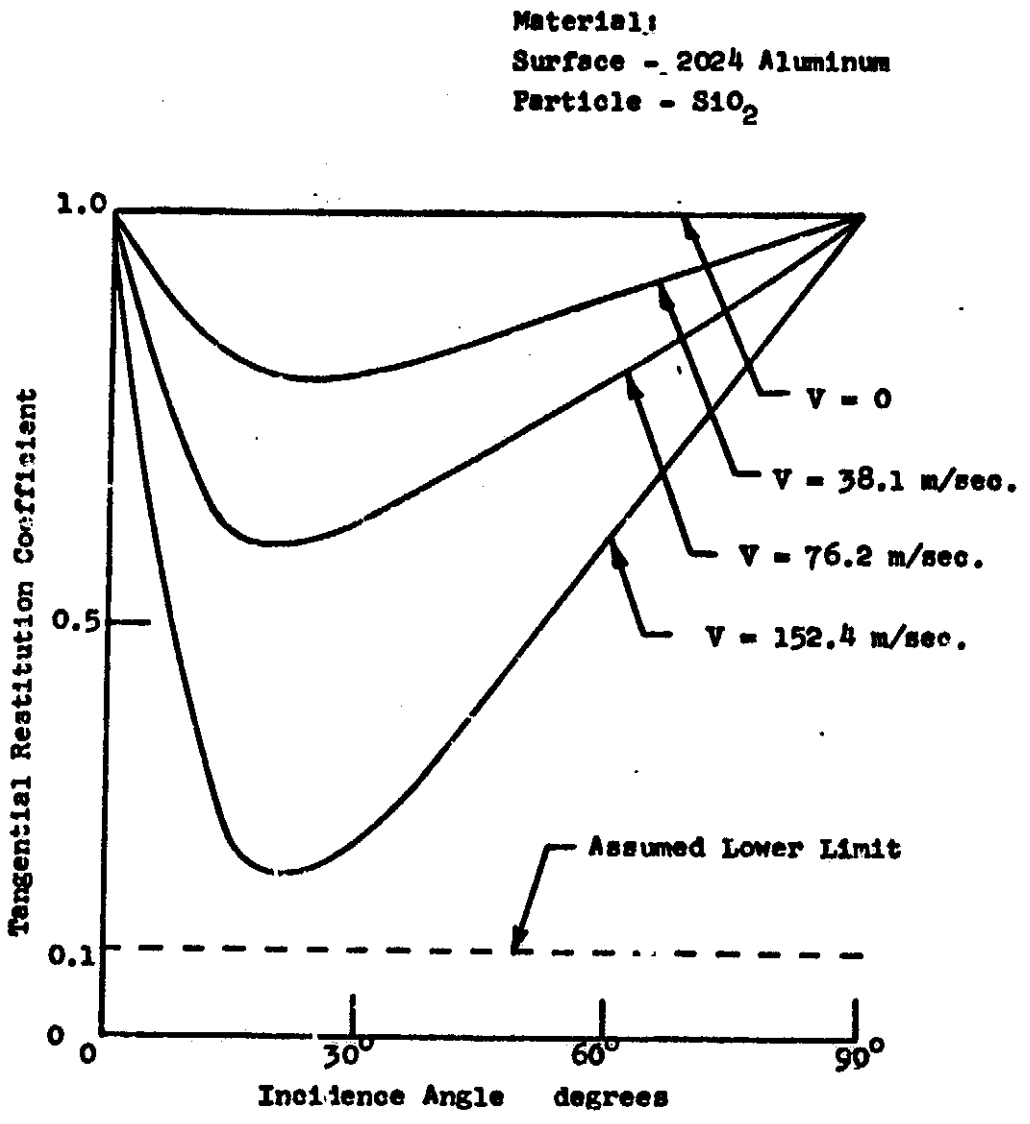


FIGURE 14. TANGENTIAL RESTITUTION COEFFICIENT

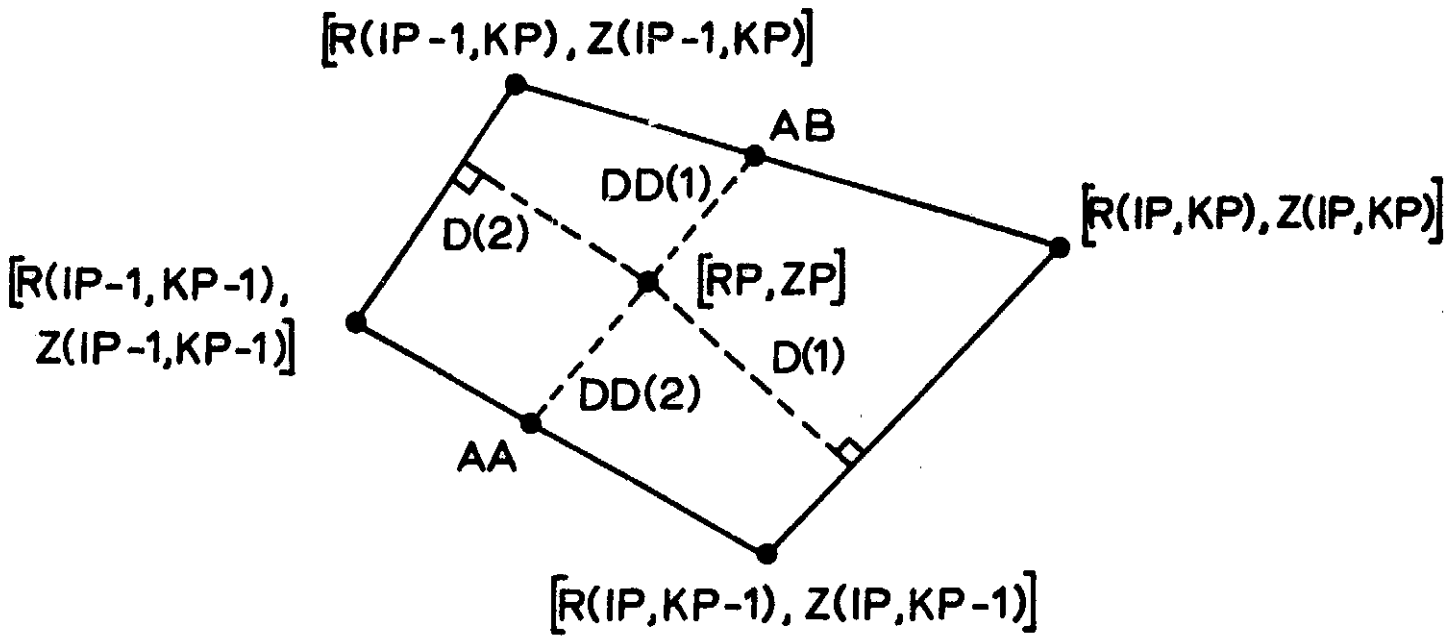


FIGURE 15. TYPICAL GRID CONFIGURATION USED IN POLATE

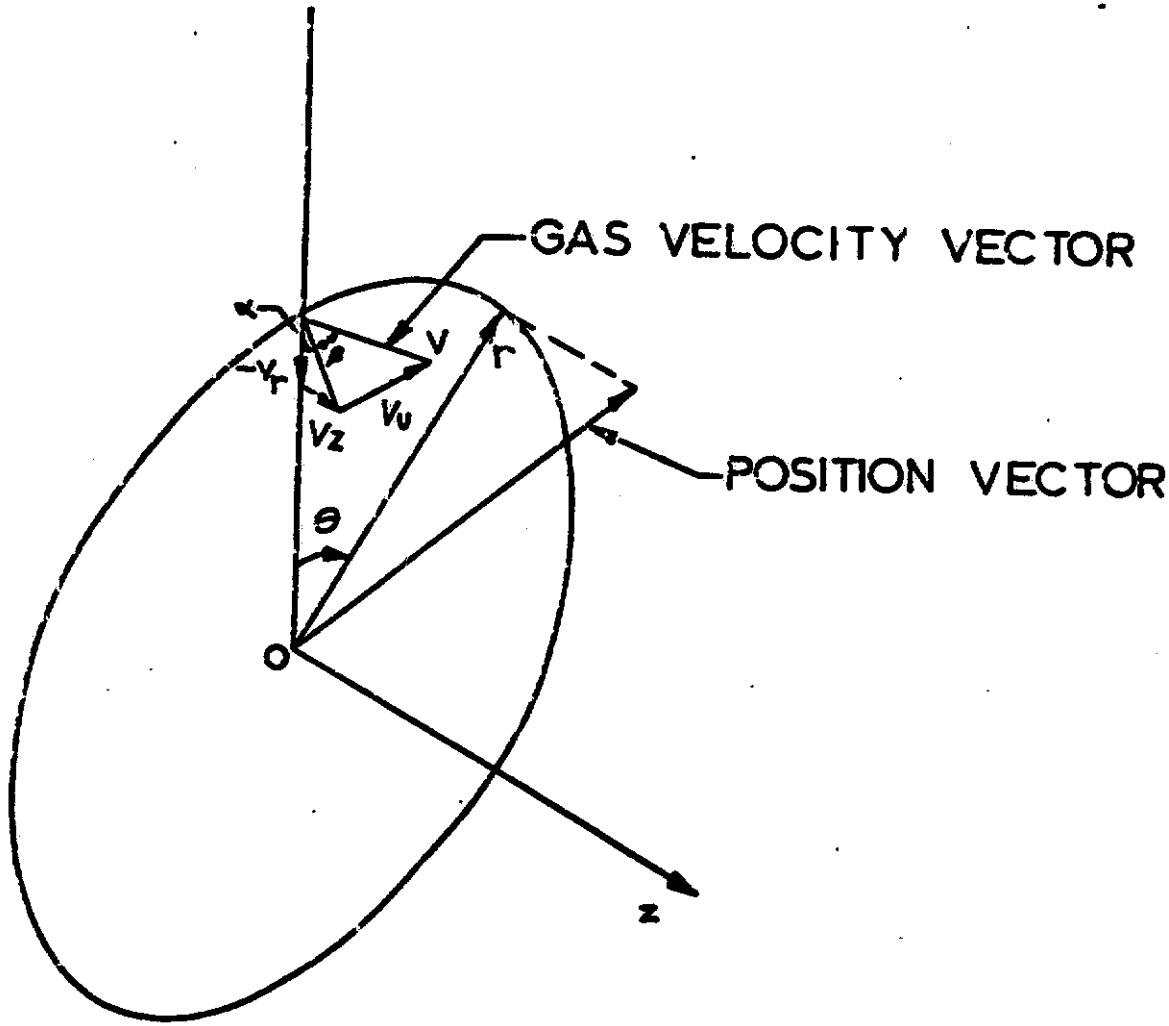


FIGURE 16. COORDINATE SYSTEM AND TYPICAL GAS VELOCITY COMPONENTS

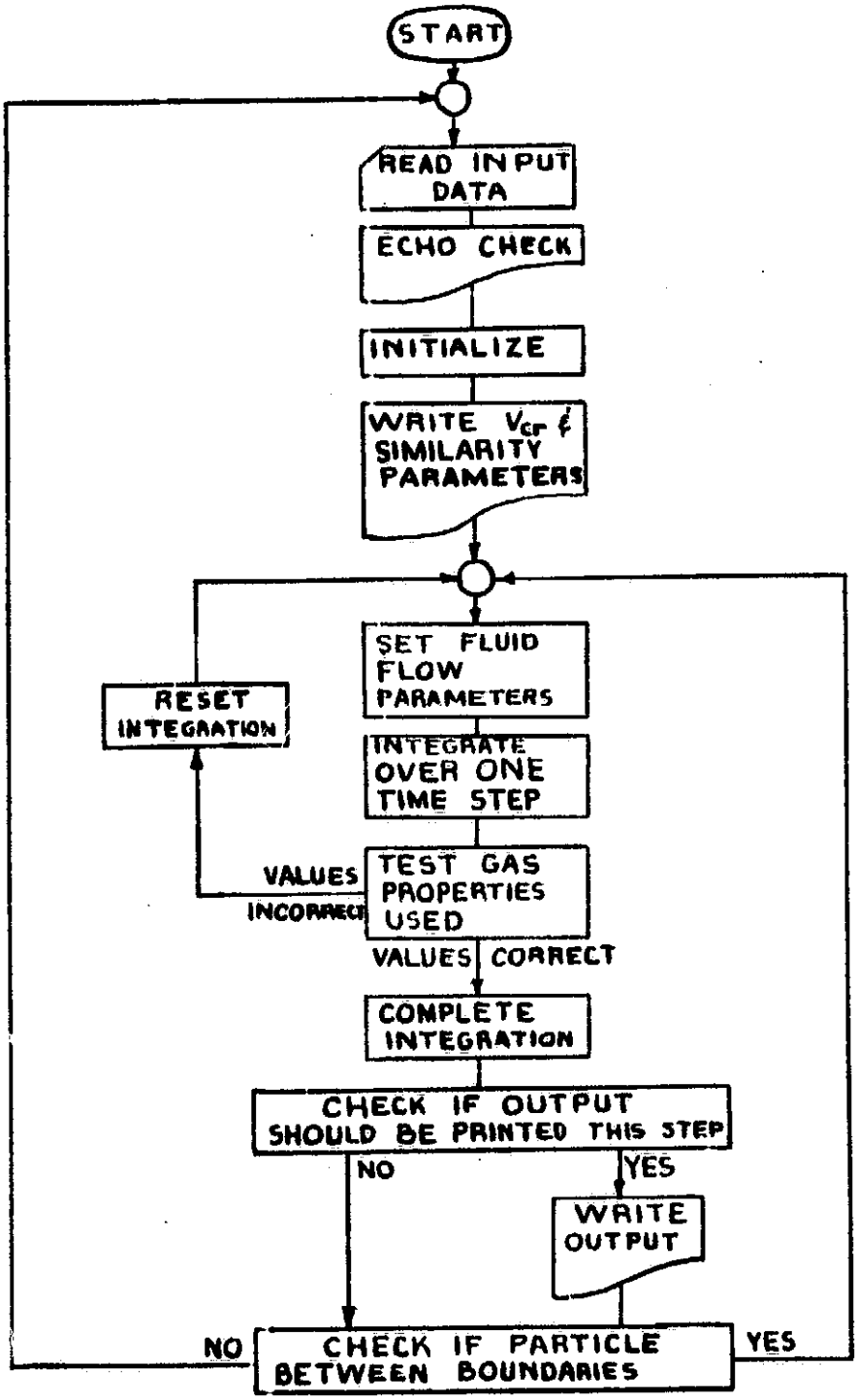


FIGURE 17. PARDIM FLOW CHART

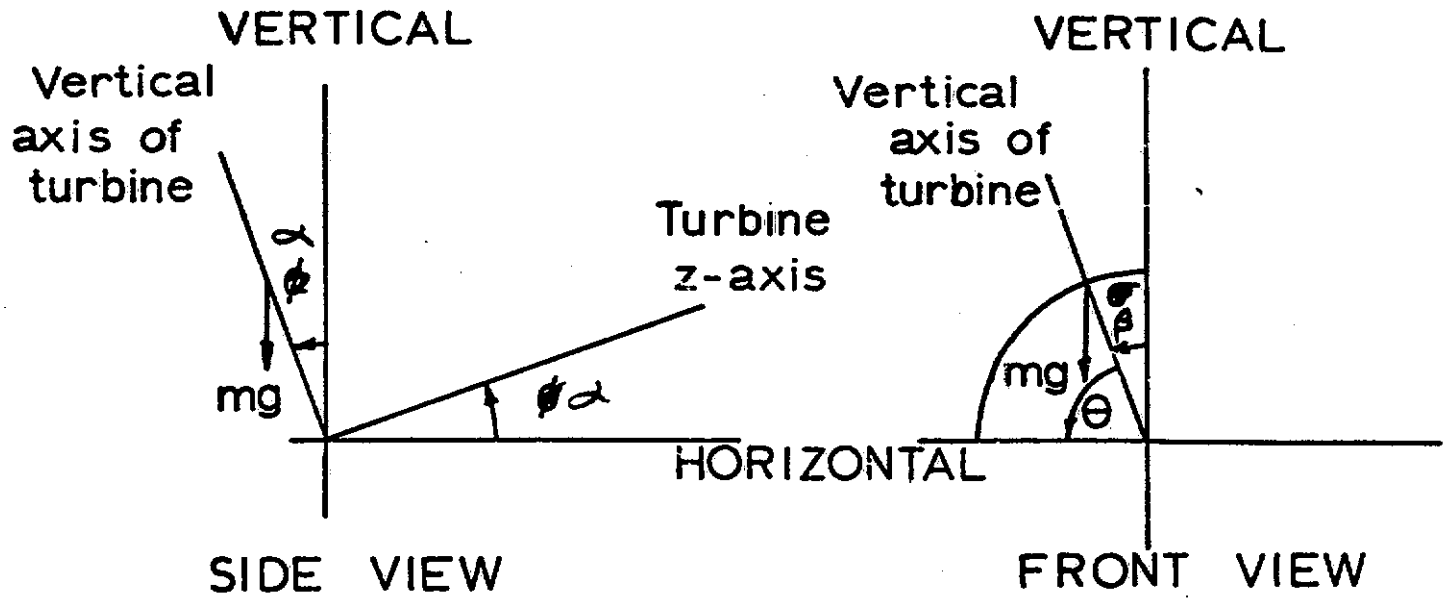


FIGURE 18. ORIENTATION COORDINATES

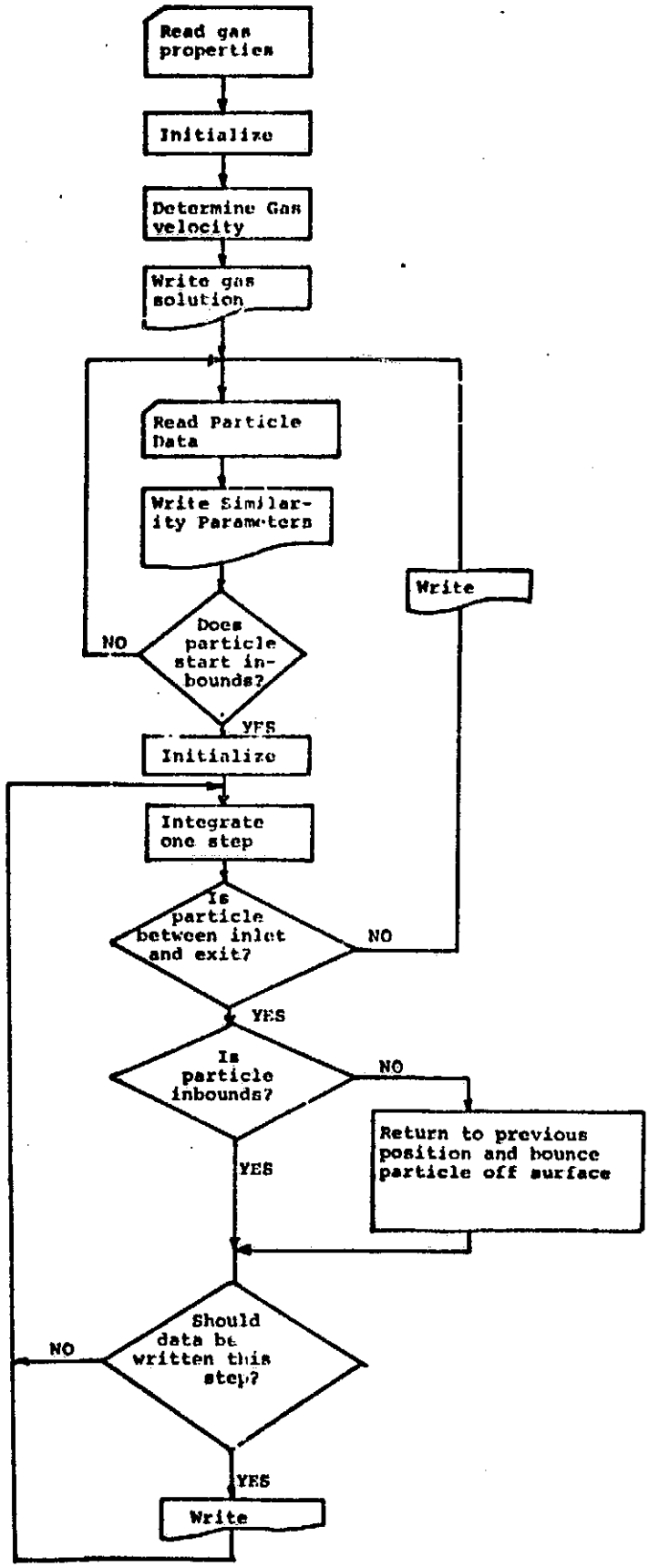


FIGURE 19. TWO-DIMENSIONAL SCROLL FLOW CHART

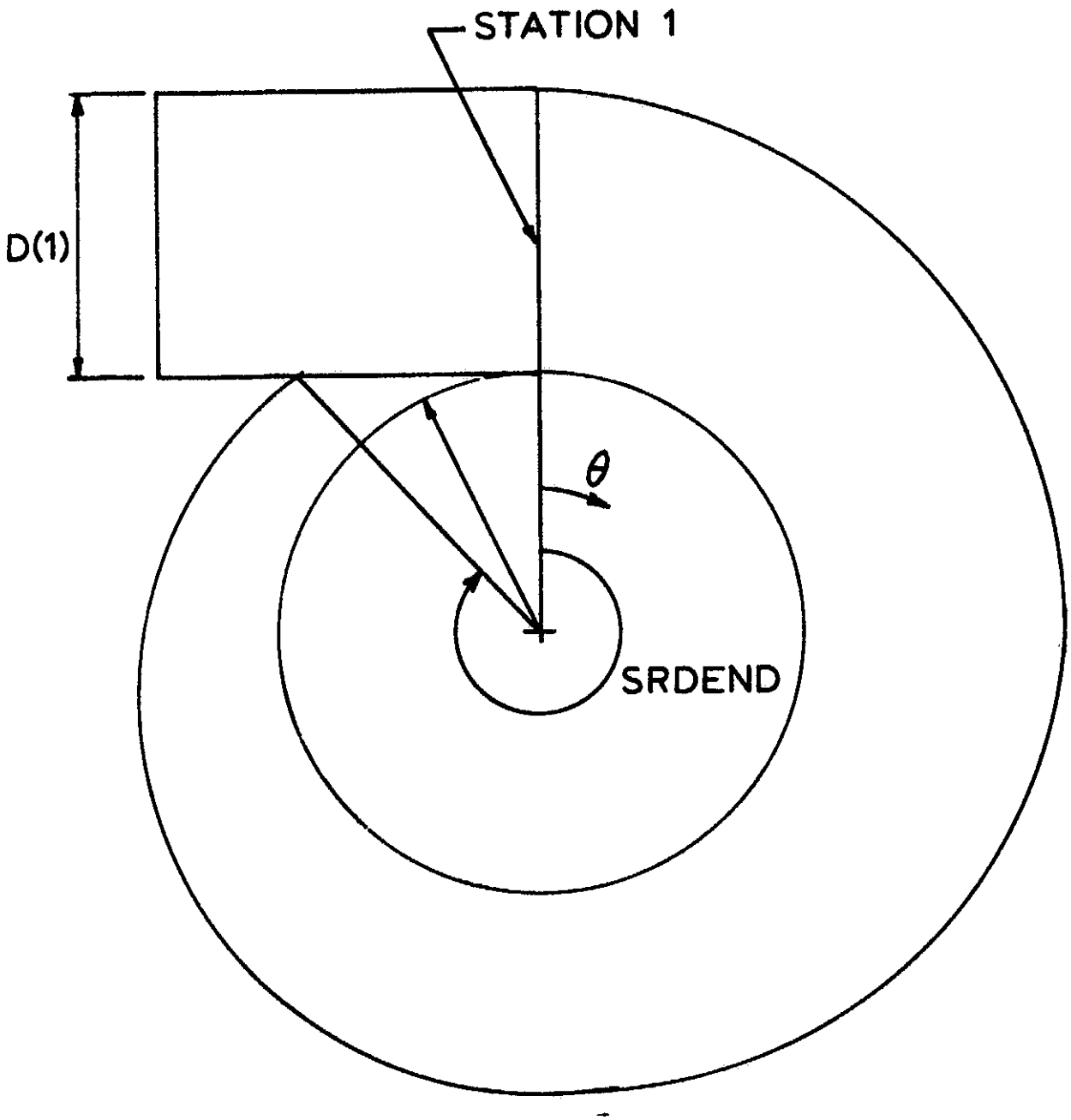


FIGURE 20. SCROLL GEOMETRY

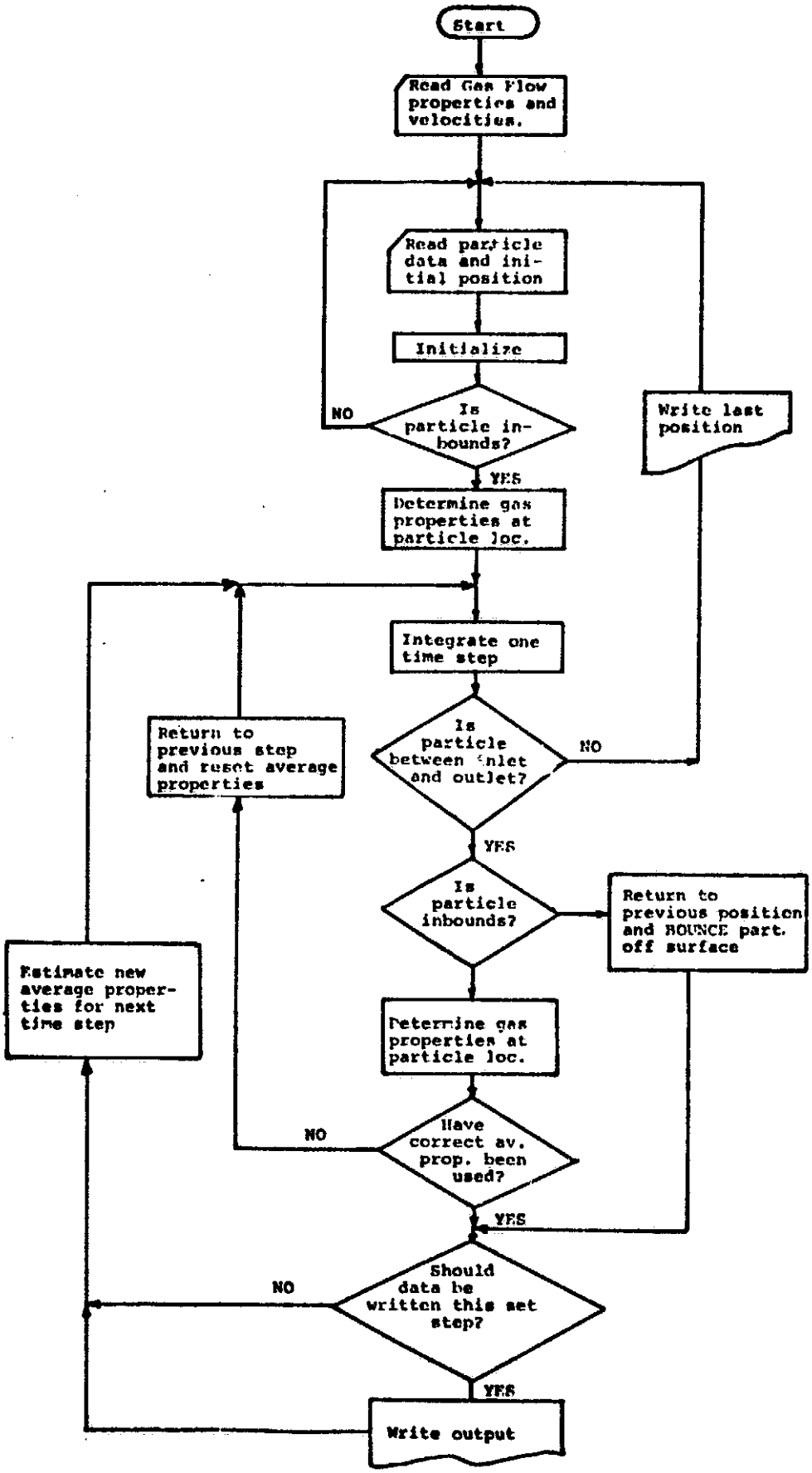
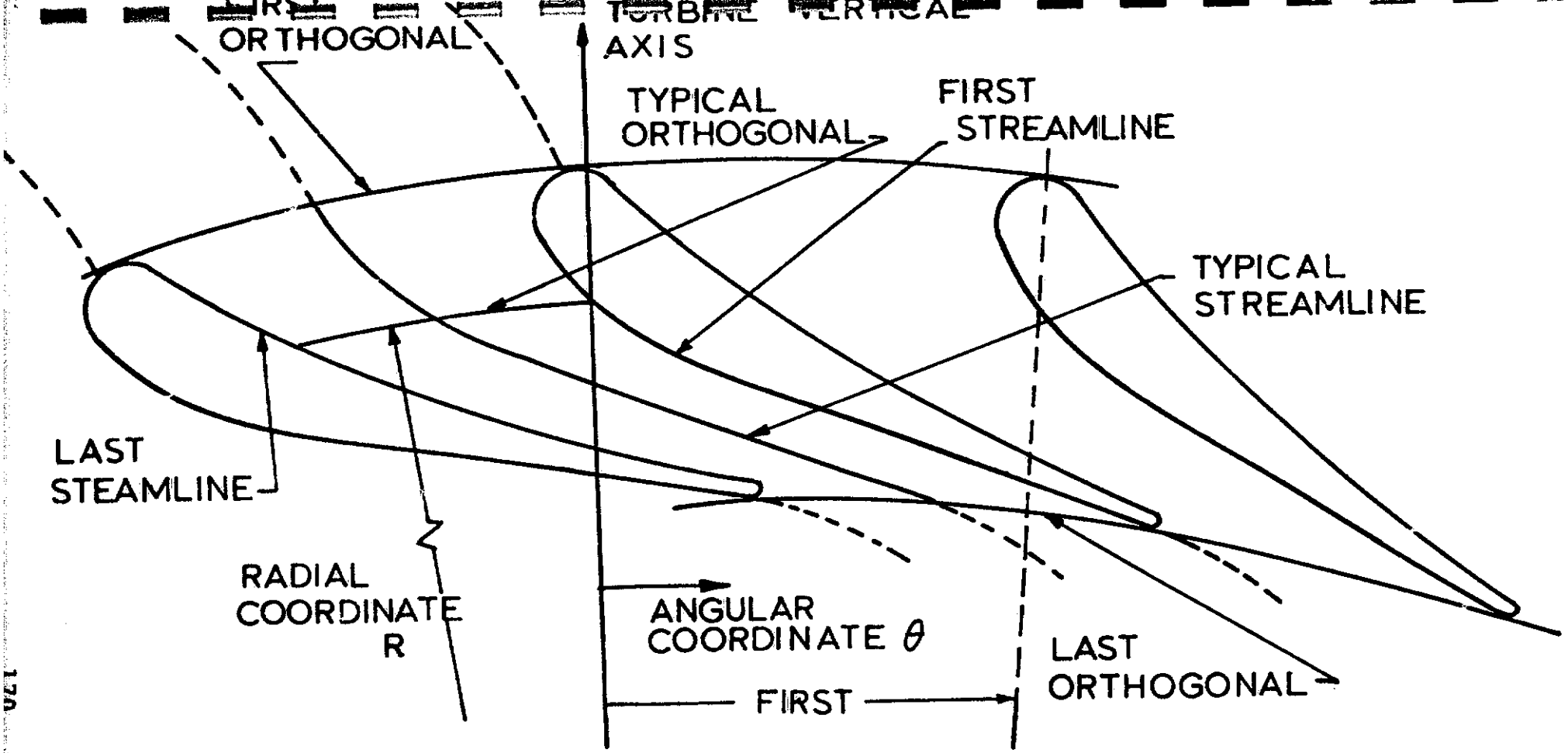


FIGURE 21. FLOW CHART FOR STATOR



NOTE: Data must be input from first to last orthogonal and from first to last streamline.

FIGURE 22. NOZZLE GEOMETRY AND COORDINATE SYSTEM USED IN STATOR PROGRAM

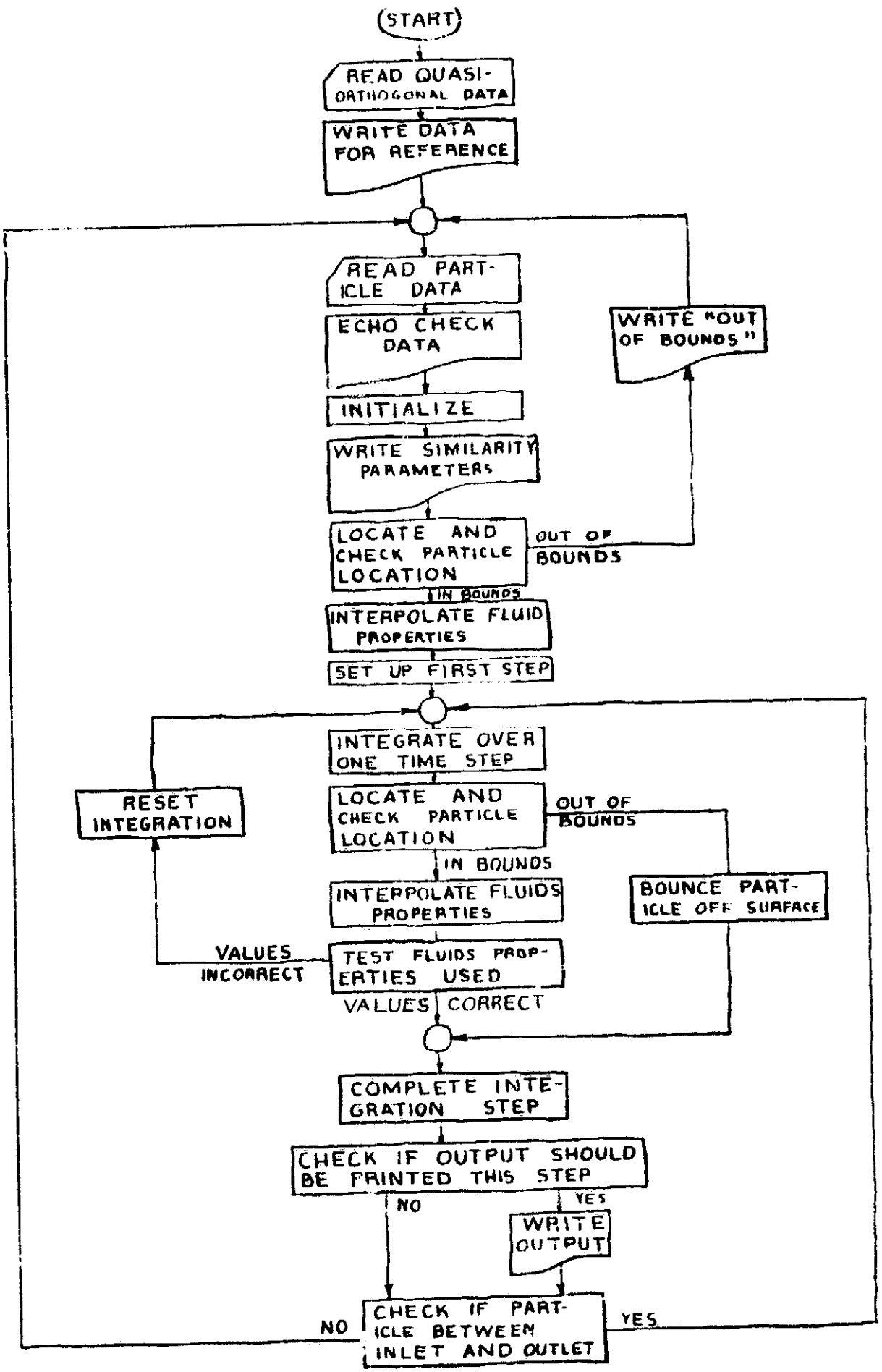


FIGURE 23. ROTOR FLOW DIAGRAM

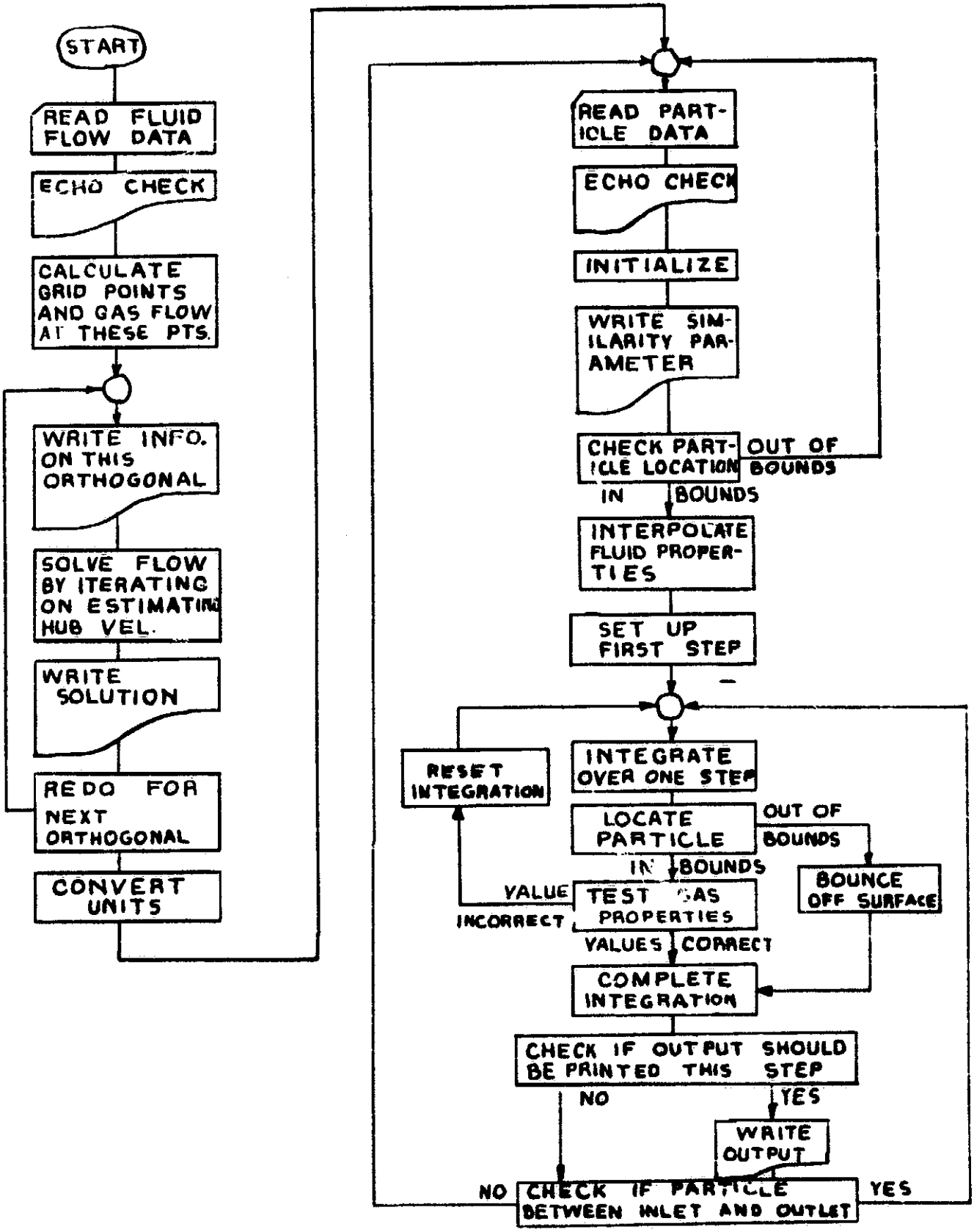


FIGURE 24. VANPY FLOW DIAGRAM

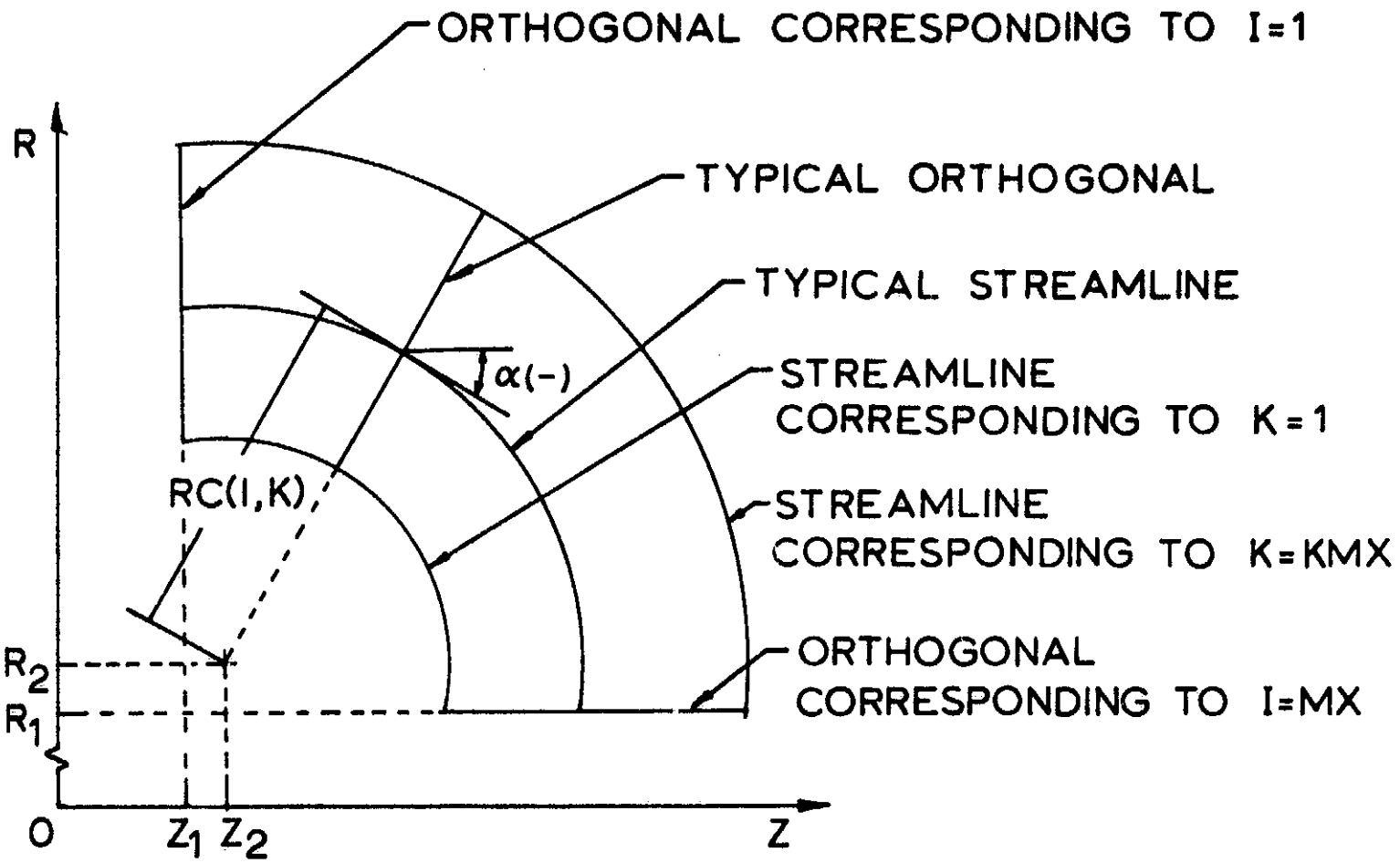


FIGURE 25. COORDINATE SYSTEM USED IN VANPY COMPUTER PROGRAM